Demand Forecasting of Sanitarywares

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ABSTRACT

The future demand of the products of an Insulator and Sanitarywares Factory is estimated by Winters' method of exponential smoothing. The estimates serve as a basis for the determination of the optimum product-mix for the various products of the factory. Optinum values of smoothing constants and their behaviour are also studied.

Introduction :

Forecating is the estimation of the future demand based on the past. Sanitarywares are consumed by the private individuals in the country. This consumption is not uniform but follows a seasonal pattern. In order to account for the seasonal variation Winters' method of exponential smoothing is applied.

Winters' method for a multiplicative seasonal pattern assumes that the time series model is

$$x_t = (a_{1t} + b_2 t)\, c_t + \varepsilon_t$$

where,

a_{1t} = the base component, usually called the permanent component

b_2 = a linear trend component

c_t = a multiplicative seasonal factor

ε_t = the usual random error component

It is assumed that the season contains L periods, and the seasonal factors are defined such that they sum to the length of the season, that is,

$$\sum_{t=1}^{L} c_t = L$$

In the multiplicative seasonal model, as the average level of the series, $a_{1t} + b_2 t$, increases, the amplitude of the seasonal pattern also increases (Fig. 1).

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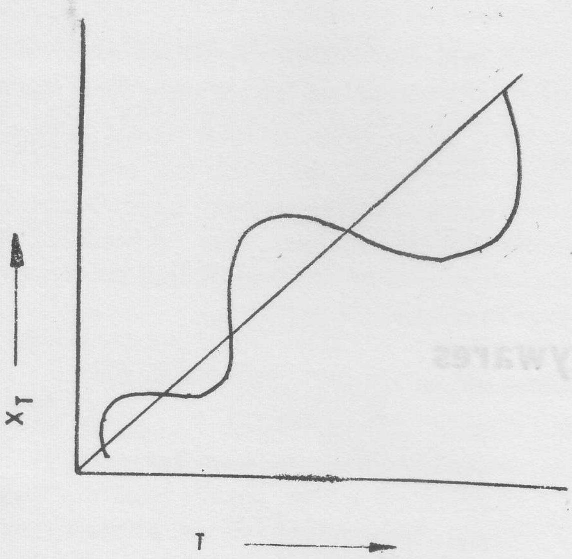

Fig. 1. Seasonal pattern with amplitude proportional to the average level of the series.

As the available sales data of the factory exhibit multiplicative seasonal variation, the above model seems to be more appropdriate.

2. Procedure :

The procedure for periodically revising estimates of the model parameters and for forecasting is the following :

At the end of any period T, after observing the realization for that period x_T,

1. Update the estimate of the permanent component :

$$a_1(T) = \alpha \frac{x_T}{c_T(T-L)} + (1-\alpha)[a_1(T-1) + b_2(T-1)]$$

where,

$0 \leq \alpha < 1$ is a smoothing constant

2. Update the estimate of the trend component

$$b_2(T) = \beta [a_1(T) - a_1(T-1)] + (1-\beta)\, b_2(T-1)$$

where,

$0 \leq \beta < 1$ is a second smoothing constant.

3. Update the estimate of the seasonal factor for period T

$$c_T(T) = \gamma \frac{x_T}{a_1(T)} + (1-\gamma)\, c_T$$

where,

$0 \leq \gamma < 1$ is a third smoothing constant.

4. To forecast the observation in any future period $T+\tau$

$$x_T + \tau(T) = [a_1(T) + b_2(T)\tau]\, c_T + \xi(T + \zeta - L)$$

The development of a forecasting system using Winters' method requires initial values of the parameters $a_1(0)$, $b_2(0)$ and $c_t(0)$ for $t = 1, 2, \ldots \ldots, L$. These are calculated from the historical data.

Fig. 2 shows the actual sales volumes, initialization phase and forecasting phase. Period-1 to 24 is the initialization phase and period-25 to 36 is the forecasting phase. In the initialization phase, observing values are used as x_T to compute a_{1t}, b_2 and c_t and in the forecasting phase, estimated values are used as x_T to compute a_{1t}, b_2 and c_t.

3. Choice of Smoothing Constant :

In any application of exponential smoothing, it is necessary to specify a value for the smoothing constant. The smoothing constants control the number of past realizations of the time series that influence the forcecast. Larger values of the smoothing constant give weight to the more recent historical data and cause the forecasting system to respond rapidly to parameter shifts. In this case, the smoothing constants Alpha, Beta and Gamma are optimized by carrying out a sequence of trials on the available 24 months' historical data using values ranging from 0 to 0. 99 for each constant. Here measure of effectiveness is the minimum sum of square errors. Sample iterations and optimm values of Alpha, Beta and Gamma are shown in Table — 1. In this table, minimum residual sum of square is 843.2084961 and its corresponding values of smoothing constants are $\alpha = 0.34$, $\beta = 0$ and $\gamma = 0$.

4. Behaviour of Smoothing Constants :

The responses of the smoothing constants α, β and γ to the residual sum of squares of the least square criterion are different. Fig. 3: shows the responses of γ and β with respect to the residual sum of squares of the least square criterion. It is noted from the figure that for the same changes in the values of γ and β, residual sum of squares changes more in case of γ. It means that for the same increase in tha values of γ and β keeping α constant, the value of γ may lead to more error in the estimation, No generalized conclusion can be drawn in this regard.

Fig. 4 : shows two curves showing the responses of α towards the residual sum of squares. It is observed from the curves that the minimum value of the residual sum of square occurs at $\alpha = 0.34$ corresponding the curve having $\beta = 0$ and $\gamma = 0$. In fact, this point refers to the optimum values of the smoothing eonstants.

Fig. 5 : shows how the permanent component varies with period for different values of α, when $\beta = 0$ and $\gamma = 0$.

5. Conclusions :

The estimates of the multiplicative seasonal model used in this paper form a basis for the determination of the optimum product-mix of the various products of the factory. The values can be safely considered as a lower bound for the production quantities. Better estimates would be obtained if a model could be developed incorporating various external demand parameters.

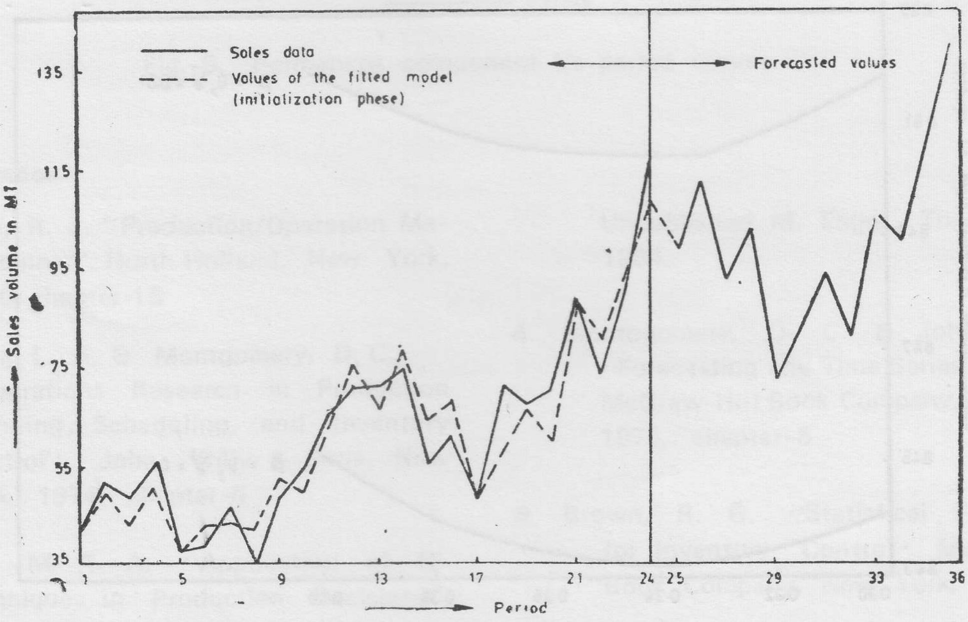

Fig. 2. Sales data, value of the fitted model and forecasted values.

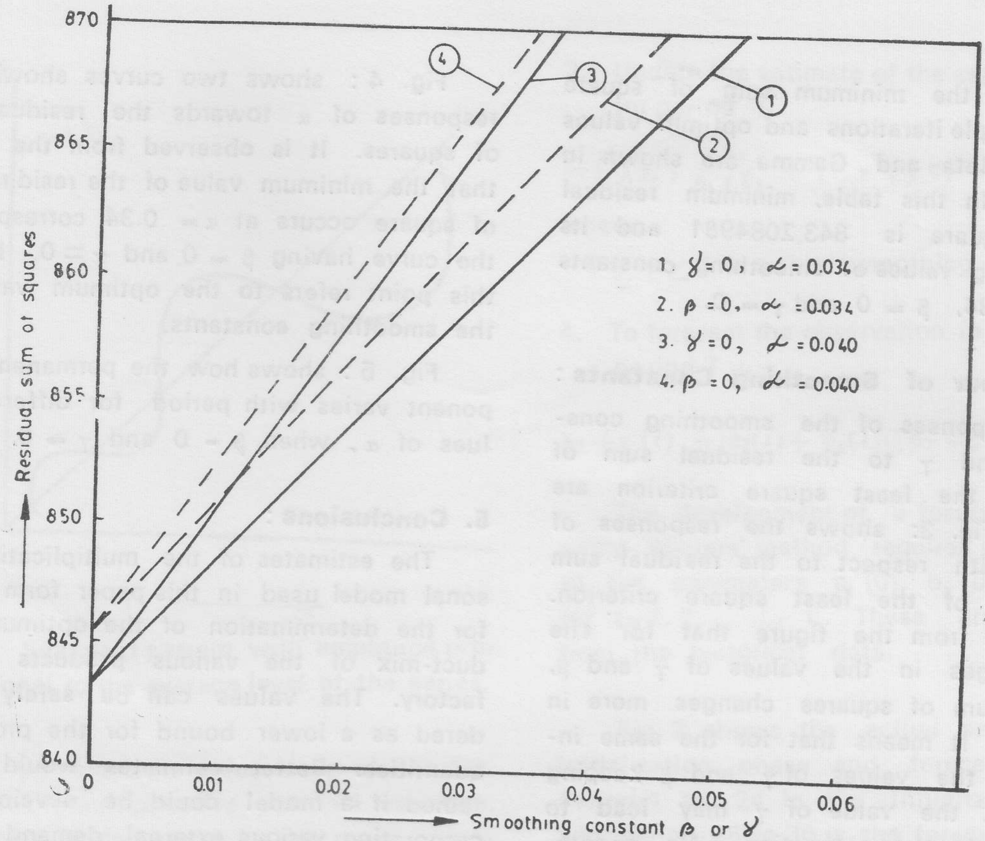

Fig. 3. Residual sum of squares Vs β or γ curves.

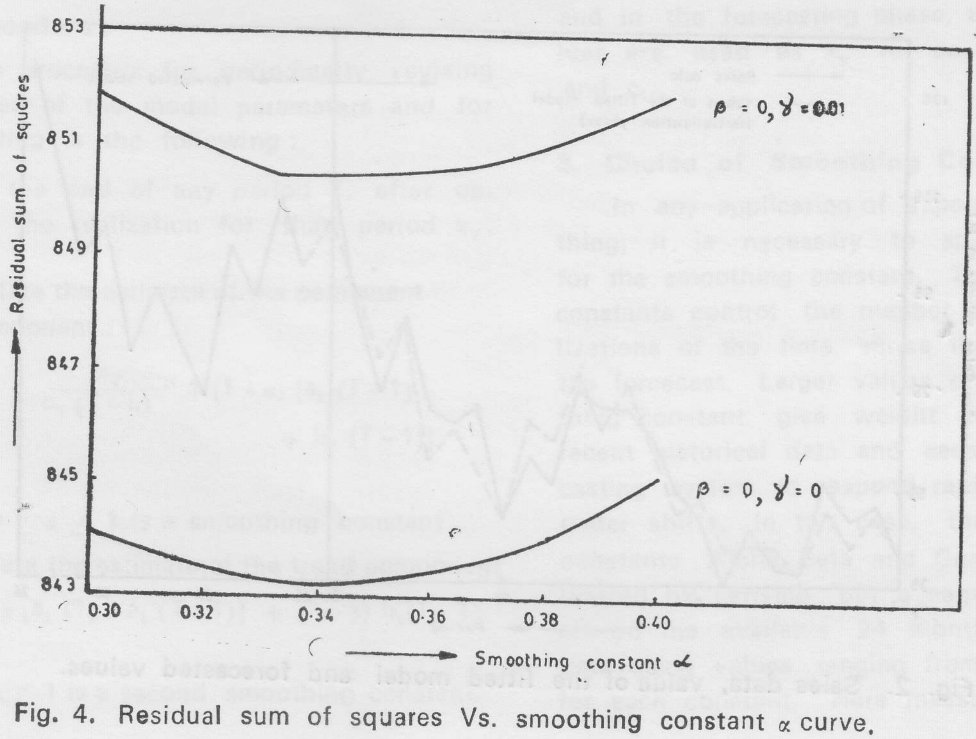

Fig. 4. Residual sum of squares Vs. smoothing constant α curve.

SMOOTHING CONSTANT OPTIMIZATION ROUTINE

Alpha	Beta	Gamma	Residual sum of Scuares
0.3300	0.0	0.0	843.2675781
0.3300	0.0	0.0100	850.682373
0.3300	0.0	0.0200	858.1572266
0.3300	0.0100	0.0	848,8520508
0.3300	0.0100	0.0100	856.2949219
0.3300	0.0100	0.0200	863.7971191
0.3300	0.0200	0.0	854,2377930
0.3300	4,0200	0.0100	861.7124023
0.3300	0.0200	0.0200	869.2465820
0.3400	0.0	0.0	843.2084961
0.3400	0.0	0.0100	850.54 0156
0.3400	0.0	0.0200	8 7.9313965
0.3400	0.0100	0.0	848.7875977
0.3400	0-0100	0.0100	856.1511230
0.3400	0.0100	0.0200	863.5708008
0.3400	0.0200	0.0	854.1791992
0.3400	0.0200	0.0 00	861.5742188
0.3400	0.0200	0.0200	869.0280762
0.3500	0.0	0.0	843.2736816
0.3500	0.0	0.0100	850.5244141
0.3500	0.0	0.0200	857.8312988
0.3500	0.0100	0.0	848.8535156
0.3500	0.0100	0.0100	856.1364746
0.3500	0.0100	0.0200	863.4750977
0.3500	0.0200	0.0	854.2578125
0.3500	0.0200	0.0100	861.5734863
0.3500	0.0200	0.0200	868.9489746

The Optimum Smoothing Constants are

Alpha = 0.3400 **Beta =** 0.0 Gamma = 0.0

Table 1. Smoothing constant optimization rautine