The Effect of a Wake on the Mixing of Two Confined Incompressible Streams

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Abstract :

Two different turbulent boundary layer velocity profiles at two adjacent nozzle exits were generated to investigate the influence of the initial conditions on the flow. The nozzles were fitted in a wind tunnel of test section, 1.5 ft x 1.5 ft x 10 ft 8 in, (45.72 cm x 45.72 cm x 325.12 cm). Two nozzles of aspect ratios 4.5 and 3 were used for the experiment. The nozzles were separated by a 0.75 inch (1.905 cm) thick wooden plank producing a wake in between the jets. The displacement thicknesses of the velocity profile at the exit planes of the two nozzles were $\delta^*/\delta = 0.128$ and 0.137 respectively with corresponding Reynolds numbers, $Re_D = 1.65 \times 10^5$ and 2.27×10^5. Based on the exit condition the boundary layer was assumed to be turbulent.

For each case the mean axial velocity and the mean static pressure were measured across the jets. The wake formed between the two interacting streams disappeared approximately at the same axial distance, $x/D = 7$ for both the nozzles. The pressure distribution was found to be uniform across the streams except for a small suction in the region of the wake on the downstream side of the thin plate which separated the two streams.

Notation :

A	aspect ratio of the rectangular nozzle ($=b/D$)
b	width of the rectangular nozzle
C_P	coefficient of pressure in the jet
D	depth of the rectangular nozzle
D_S	depth of the upper channel
D_T	depth of the test section
H	shape factor at the nozzle exit boundary layer
h	height of the step
n	power of the velocity profile
Re_D	Reynolds number based on the average velocity and the nozzle depth.
U_J	mean axial velocity of the jet
U_{Jmax}	maximum mean axial velocity of the jet
U_S	mean axial velocity of the upper stream
U_{Smax}	maximum mean axial velocity of the upper stream
U_{SN}	mean axial velocity in the upper stream at the exit plane
U_{SNmax}	maximum mean axial velocity in the upper stream at the exit plane
U_N	mean axial velocity at the nozzle exit boundary layer
U_{Nmax}	maximum mean axial velocity at the nozzle exit boundary layer

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x	axial distance measured from the nozzle exit
y	cross stream ordinate measured from the bottom wall of the test section
y_m	minimum value of y where $U_J = U_{Jmax}$
Y	cross stream ordinate measured from the bottom wall of the nozzle
Y_S	upper stream ordinate measured from the top of the nozzle at the exit plane

Greek Symbols :

δ	boundary layer thickness at the nozzle exit
δ^*	displacement thickness at the nozzle exit
η	non-dimensional co-ordinate perpendicular to the wall ($= y/\delta$)
θ	momentum thickness at the exit of the nozzle
ν	kinematic coefficient of viscosity of air
ρ	density of air
$\triangle x$	distance between the centres of two successive measuring holes
$\triangle U$	difference of mean axial velocity between the jet and the upper stream ($= U_J - U_{smax}$)
$\triangle U_{max}$	maximum local excess mean axial velocity ($= U_{Jmax} - U_{Smax}$)

Introduction :

When a jet of fluid emanates from a narrow slot and impinges onto a rigid wall at an angle from 0 to 90 degrees it is called a wall jet. The spread of the wall jet is inhibited on one side by the presence of a solid surface and the velocity is zero on that surface. In most practical examples, the wall jet will be turbulent. The first theory of the wall jet, laminar and turbulent, radial and plane, was developed by Glauert (1) in 1956. His basic assumption was that since the wall jet comprised of a boundary layer flow near the wall and a free mixing flow in the outer part, there could not be a unique solution for the flow as a whole. Glauert (1) used an eddy viscosity distribution consistent with the power law velocity profile in the inner layer. On the other hand, a constant eddy viscosity (appropriate to a free jet) was used across the outer layer. Glauert's (1) two-layer model for the wall jet flow is treated as classic, and has been successfully applied in many cases, even when the wall was curved and when the assumptions made in its derivation did not strictly apply.

A more practical situation would be a wall jet issuing into a stream moving in the same direction. The essential feature of the velocity profile for such a flow is that the velocity does no longer become zero at a large distance from the wall but approaches a finite value, U_s, which is in general smaller than the maximum value reached somewhere in the jet. The simplest approximation to the spreading of a jet in a moving stream is obtained by superposing the external flow with velocity, U_s, on the jet flow into still air. It is then assumed that for the same velocity difference ($U_J - U_s$) the mixing process is also the same. A consistent method involves a stretching of the streamwise co-ordinates, to take account of the different distances which fluid particles travel in unit time in the two cases. This may be regarded as a transformation from a fixed co-ordinate system in still air to a moving one. This method gives adequate answers for practical purposes in simple cases, and has been used by Kruka and Eskinazi (2) for the solution of wall jets in a moving stream. Escudier and Nicoll (3) developed integral methods for wall jets in pressure gradients where the mean velocity profile was built up by the superposition of a jet component and the logarithmic law of the wall. The velocity profile at the edge of the flow gave a skin friction law. The integral momentum equation provided a second equation, and the third, required to close the solution, was obtained by relating the non-dimensional rate of entrainment to the profile. Gartshore and Newman(4) have developed an integral method for predicting the growth and separation of a simple wall jet. Newman et al (5) have studied both theoretically and experimentally an incompressible three-dimensional turbulent wall jet originating from a circular orifice located adjacent to a plane wall. Narayan and Narasimha(6) have carried out a parametric analysis of turbulent wall jets. They have proposed that the fully developed state of the flow is governed by the total momentum flux at the slot exit, rather than by the jet velocity and the slot depth separately. Rajaratnam and Stalker(7) have carried out an experimental study on the mixing and diffusion of circular turbulent wall jets in coflowing open channel streams of water, with the ratio of the jet velocity to channel velocity varying from 2 to 30, and the depth of the flow being 10 to 30 times the jet diameter. The study is limited to jets with the same density as that of the coflowing stream and the bed of the channel has been kept nonerodible.

Mathews and Whitelaw(8) investigated experimentally and theoretically the flow in the mixing region of two jets when there is a step on the wall side and the lip is of finite thickness producing a recirculating flow and a wake in the mixing region of the jets.

Existing theoretical methods are not yet powerful enough for practical purposes and hence experimental data must be relied upon to a considerable extent. The present experimental investigation is on the interaction of an incompressible turbulent confined wall jet and a stream with a wake existing between the two.

The knowledge of such mean flow properties of turbulent wall jets in the presence of wakes is useful in many physical applications such as the jet flaps, the jet flow below a sumberged sluice gate, the discharge of effluents from a large pipe buried in the riverbed through a series of short pipes or nozzles as coflowing circular wall jets, the flow behind a ship hull while sailing in a sea etc.

The Experimental Set-Up and Experiments :

A subsonic wind tunnel with a test section, 1.5 ft. x 1.5 ft x 10 ft 8 in. (45.72 cm x 45.72 cm x 325.12 cm) was used for the experiment. The details of the wind tunnel were presented in reference (9). The test section was divided into two halves. The upper half always having a lower average velocity than the lower half, was considered as the main stream. The lower half was treated as the nozzle and had always a higher average velocity than the main stream. Such nozzles of aspect ratios 4.5 and 3 were set up within the test section of the wind tunnel as shown in figure 1. Air was allowed to flow through the nozzle and through the upper portion of the duct with a 0.75 in. (1.905 cm) thick plate in between them. This was considered as the wake generator. At the exit section of the nozzle one step of height 1 in. (2.54 cm) was built up and the jet spreaded over the lower plate. The jet expanded on the upper side against another

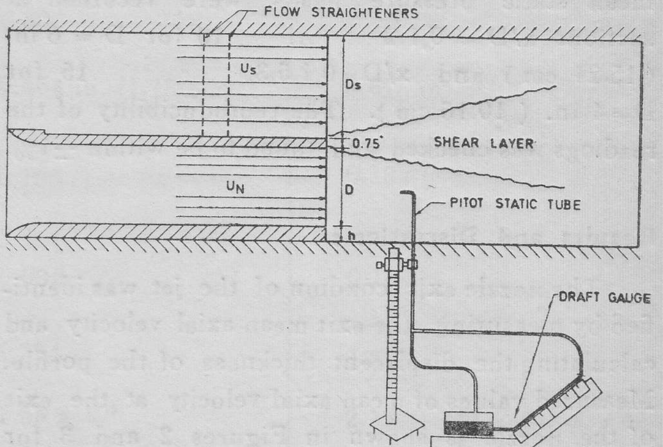

Fig. 1. Schematic diagram of the experimental set-up

stream of air flowing over the nozzle. This wall jet, confined by the wall of the test section was allowed to grow in the downward direction over a perspex floor as shown in Figure 1. On the bottom of the duct eleven measuring holes were made at interval of 6 inches (15.24 cm) which corresponds to $\Delta x/D = 1$ for $D=6$ inches (15.24 cm) and $\Delta x/D=1.5$ for $D=4$ inches (10.16 cm), where Δx is the distance between the successive holes and D is the nozzle depth. The holes were circular and each hole was 0.25 inch in diameter, which permitted the traversing of the Pitot-static tube through them. The Pitot-static tube was traversed vertically up and down by rack and pinion arrangement over a stand graduated to read 0.01 inch. The United Sensor Pitot-static tube of outer diameter 1/16th inch (0.159 cm) was used for the measurement of mean velocity. The Pitot-static tube was connected to an Ellison draft gauge. Both the mean axial velocity and the mean static pressure heads were recorded at sections $x/D = 0,1,2 \ldots \ldots \ldots 10$ for $D = 6$ in. (15.24 cm) and $x/D=0,1.5,3, \ldots \ldots \ldots 15$ for $D=4$ in. (10.16 cm). The reproducibility of the readings was checked and found to be within ±1%.

Results and Discussion :

The nozzle exit condion of the jet was identified by measuring the exit mean axial velocity and calculating the displacent thickness of the porfile. Measured values of mean axial velocity at the exit of the nozzle is shown in Figures 2 and 3 for nozzle depths, D=4 in. (10.16 cm) and 6 in. (15.24 cm) respectively. The velocity profile is found to be symmetrical about the nozzle centreline. The experimental points were fitted to the equation, $U_N/U_{N\max} = (Y/\delta)^{1/n}$, and the value of n was computed by the least squares principle. The curves corresponding to the equation, $U_N/U_{N\max} = (Y/\delta)^{1/n}$ are also given in Figures 2 and 3 to show the agreement with experimental points. The boundary lsyer thickness was obtained by plotting the experimental values of mean axial velocity at different distances from the wall and then measuring the distance for which $U_N/U_{N\max} = 0.99$. The displacement thickness, the momentum thickness and the shape factor of the velocity profile were calculated by using equations(1), (2) and (3) respectively.

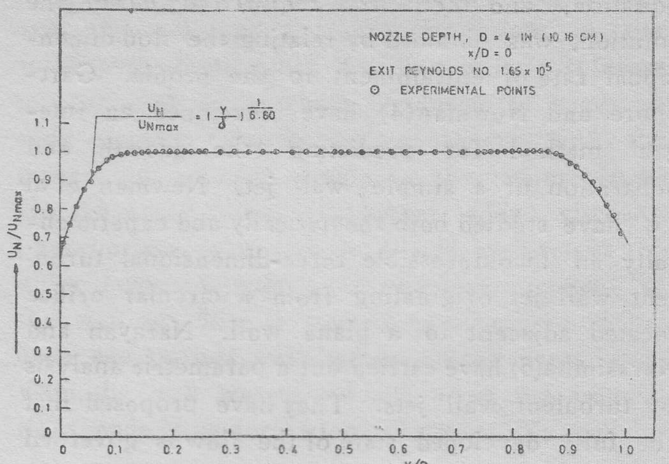

Fig. 2. Measured mean axial velocity distribution in the nozzle at the exit plane.

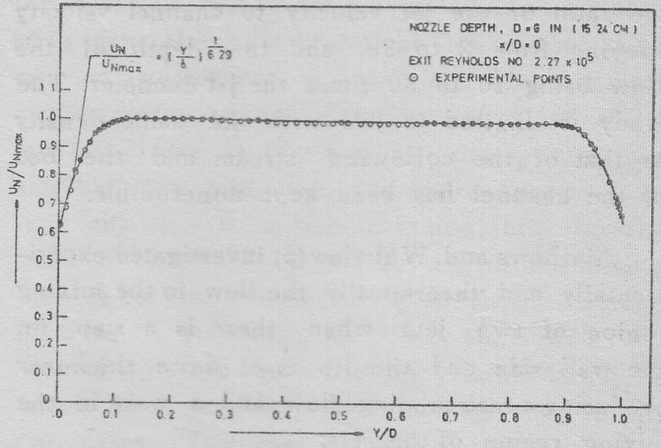

Fig. 3. Measured mean axial velocity distribution in the nozzle at the exit plane.

$$\delta^*/\delta = \int_0^1 \left(1 - \frac{U_N}{U_{N\max}}\right) d(y/\delta) \qquad (1)$$

$$\theta/\delta = \int_0^1 \frac{U_N}{U_{N\max}} \left(1 - \frac{U_N}{U_{N\max}}\right) d(y/\delta) \qquad (2)$$

$$H = \delta^*/\theta \qquad (3)$$

The area under the velocity profile at the exit plane was integrated by using Simpson's rule and subsequently the area-average velocity at the exit was computed dividing the integrated area by the cross sectional area of the nozzle exit. The Reynolds number, Re_D, was calculated on the basis of the area-average axial velocity and the nozzle depth, D. The computed values of the parameters which identify the exit conditions are given in Table 1. Considering all these characteristics of the velocity profile, the boundary layer at the nozzle exit was assumed to be turbulent.

TABLE 1 : The Nozzle Exit Conditions

Parameter	Values	
Depth of the nozzle, D	4 in. (10.16 cm)	6 in. (15.24 cm)
Power of the velocity profile, n	6.80	6.29
Boundary layer thickness, δ	0.464 in. (1.18 cm)	0.618 in. (1.57 cm)
Ratio of displacement thickness to boundary layer thickness, δ^*/δ	0.128	0.137
Ratio of momentum thickness to boundary layer thickness, θ/δ	0.099	0.104
Shape factor, H	1.294	1.318
Reynolds number, Re_D	1.65×10^5	2.27×10^5

The jet issuing from the nozzle is obstructed by the wind tunnel test section floor on the bottom side and spreads on the top side where another stream with a relatively lower average velocity exists. The velocity profile of the stream above the jet is presented in Figure 4 for nozzle depth, D=4 inches (10.16 cm). The velocity profile of the upper stream is approximately flat with a small boundary layer at both the walls. The velocity profile of the main stream for nozzle depth, D=6 inches (15.24 cm) was presented in reference (10), and it was found to exhibit similar characteristics. The oncoming flow from the nozzle interacts with the upper stream.

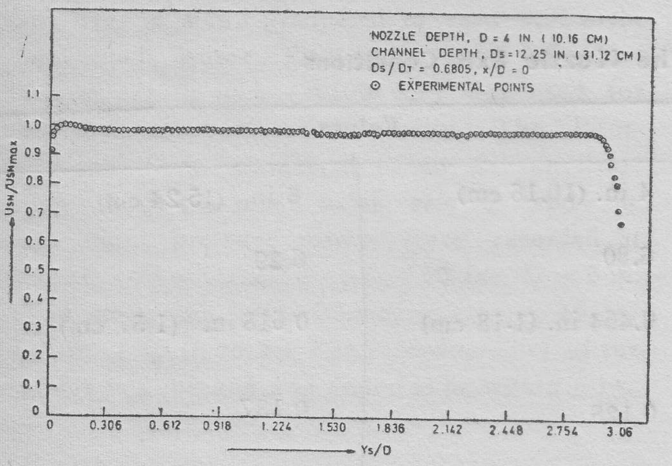

Fig. 4. Measured mean axial velocity distribution in the superimposing channel at the exit plane.

It is known that the two streams emanate from the exit with different average axial velocities, viz the lower jet having a higher average velocity than the upper one. The excess mean axial velocity is obtained by subtracting the maximum mean axial velocity of the upper stream, Usmax, from the mean axial velocity of the lower jet, U_J. The change of such an excess mean axial velocity (U_J -Usmax) along the axial direction is a matter of significance for studying the interaction between the two streams.

Figures 5 A, B and 6 A, B represent the distribution of measured excess mean axial velocity at different axial distances for nozzle depths, D = 4 in. (10.16 cm) and 6 in. (15.24 cm) respectively. The formation of a wake in the region of mixing of the two streams is very clearly observed in these figures. The wake is formed on the downstream side of the plate of thickness 0.75 inch (1.905 cm) and the size of the wake is approximately the same as the plate thickness. The excess velocity in the wake is

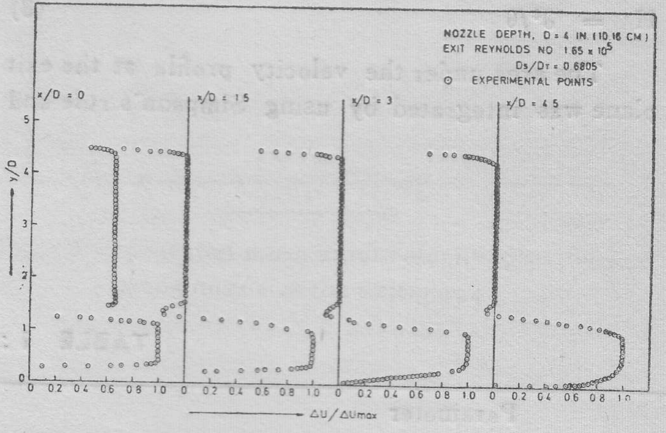

Fig. 5A. Measured mean excess velocity distribution in the jet at different axial distances.

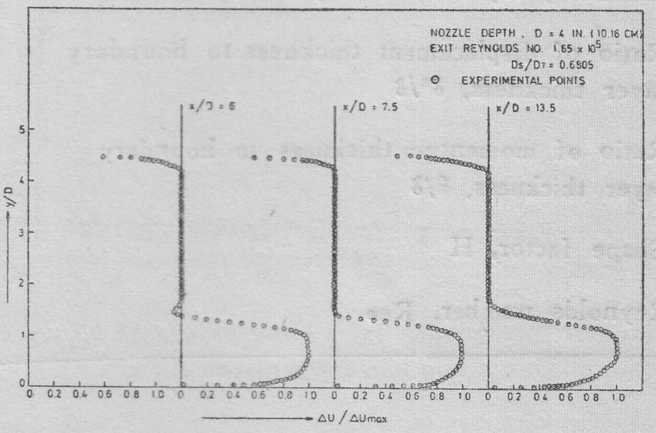

Fig. 5B. Measured mean excess velocity distribution in the jet at different axial distances.

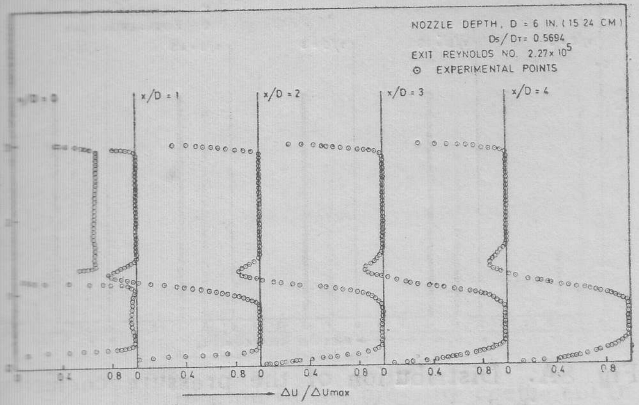

Fig. 6A. Measured mean excess velocity distribution in the jet at different axial distances.

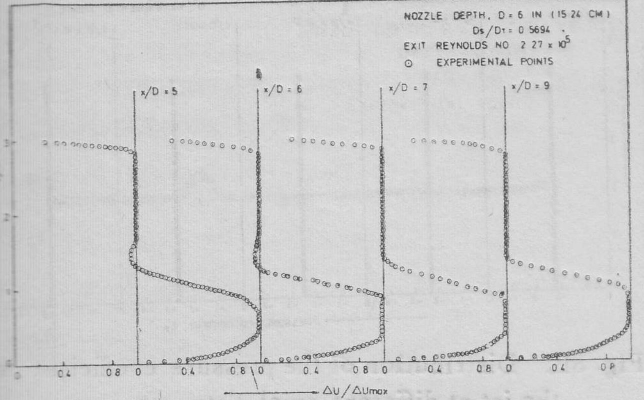

Fig 6B. Measured mean excess velocity distribution in the jet at different axial distances.

less than zero which implies that it would absorb energy from its either side. This is one of the reasons why the average velocity decays in both the upper and lower streams. The wake size close to the nozzle exit is sufficiently big to affect the flow but it gradually decreases and then disappears at an axial distance of $x/D = 7.5$ for $D = 4$ inches (10.16 cm), and $x/D = 7$ for $D = 6$ inches (15.24 cm). As the wake disappears, the interaction between the upper and the lower streams becomes more prominent. In this region the slope of the velocity profile decreases at a faster rate which implies a higher exchange of energy between the two streams. It is also observed that for both the cases the wake disappears at an approximately same axial distance of $x/D = 7$. For both the cases the boundary layer thickness at the nozzle exit is approximately the same as shown in Table 1. It is also observed that the mean excess velocities at the exit for both the cases are approximately the same. So the disappearance of the wakes at the same axial distance, $x/D = 7$, seems to be quite reasonable, although the exit Reynolds numbers are different.

There was a recirculating flow at the step corner beneath the jet. Hence the mean axial velocity head could not be recorded with the Pitot-static tube in front of the step close to the nozzle exit. The recirculating flow, however, disappears in the downstream where the boundary layer is attached to the bottom wall of the test section. It is seen from Figures 5 A as well as 6 A that the attachment of the boundary layer took place at an axial distance of $6 < x/h < 12$, which is in conformity with the experimental results of Mantle (11) who observed that the attachment of the boundary layer over discrete roughness took place approximately at a distance of $x/h = 7.5$.

The minimum distance from the wall at which the maximum velocity is attained is called the width for maximum velocity, y_m, of the jet. The experimental values of the width for maximum velocity were plotted in Figure 7. It is seen that the width for maximum velocity increases linearly along the axial direction for both the nozzle depths, $D = 4$ and 6 inches (10.16 cm and 15.24 cm). A straight line was fitted to the experimental points by the method of least squares and equations (4) and (5) were obtained for

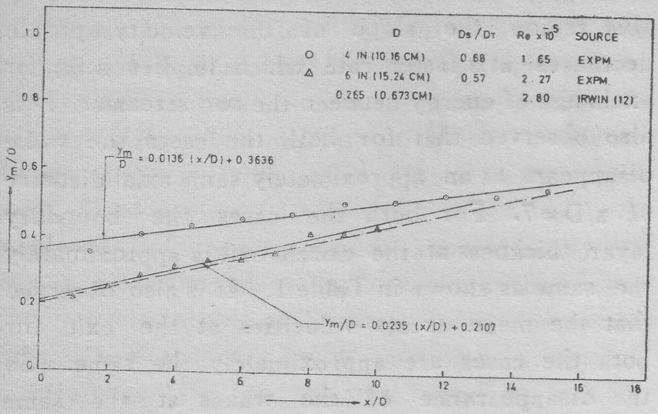

Fig. 7. Variation of the width for maximum velocity along the axial direction.

D = 4 in. (10.16 cm) and 6 in. (15.24 cm) respectively.

$$y_m/D = 0.0136(x/D) + 0.3636 \qquad (4)$$

$$y_m/D = 0.0235\, x/D) + 0.2107 \qquad (5)$$

The present experimental results are compared with those of Irwin (12) who has confirmed linear variation of the width for maximum velocity. In a similar experiment of two dissimilar moving streams Beguier (13) also found that the position of maximum velocity shifts linearly from the geometric axis of the stream.

The mean static pressure has been measured at different axial distances, and subsequently the coefficient of pressure, Cp, has been computed. The experimental values of the pressure coefficient accross the streams are plotted in Figures 8A, B and 9A, B for nozzle depths, D = 4 in. (10.16 cm) and 6 in. (15.24 cm) respectively. The distribution of the pressure coefficient is found to be approximately uniform across the jets at all axial distances except through the wake although this is not clear from Figures 8A, B and 9A, B which are drawn to a compressed scale. This is, however, clear from Figure 10 drawn to an enlarged scale. There is a small pressure drop in the region of the wake. Such static suction disappears at an axial distance, x/D = 7, where the wake no longer exists.

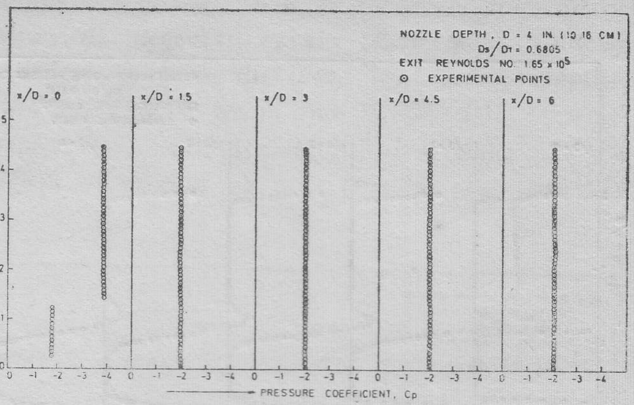

Fig. 8A. Distribution of the pressure coefficient in the jet at different axial distances.

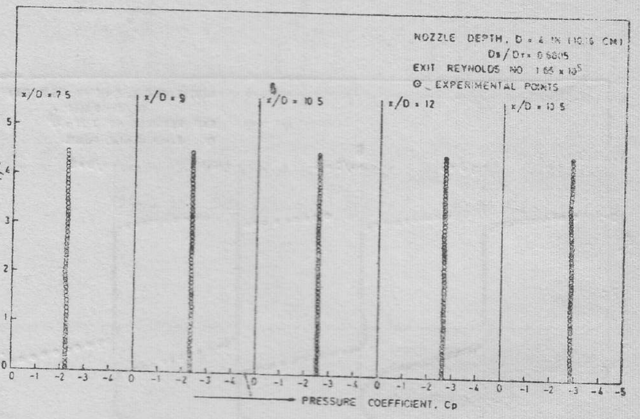

Fig. 8B. Distribution of the pressure coefficient in the jet at different axial distances.

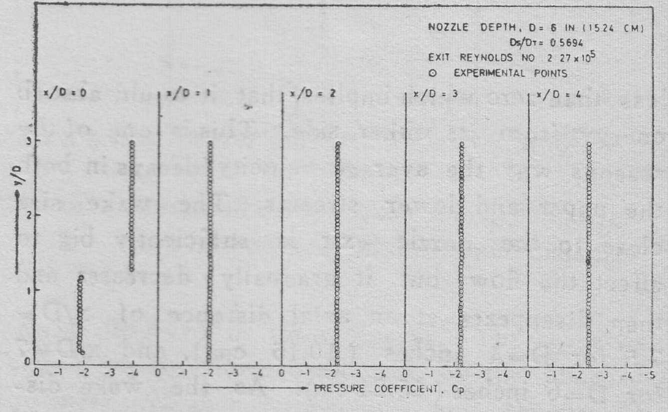

Fig. 9A. Distribution of the pressure coefficient in the jet at different axial distances.

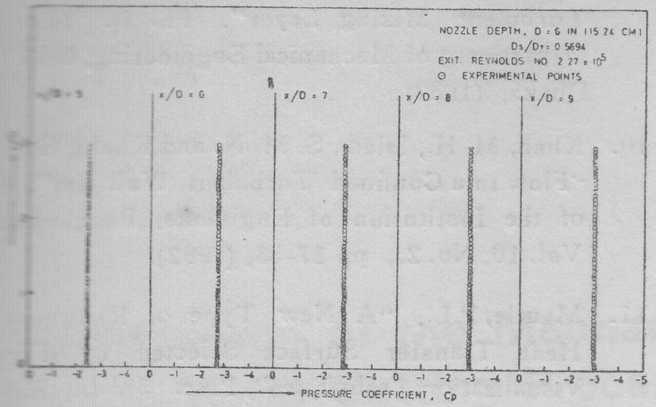

Fig. 9B. Distribution of the pressure coefficient in the jet at different axial distances.

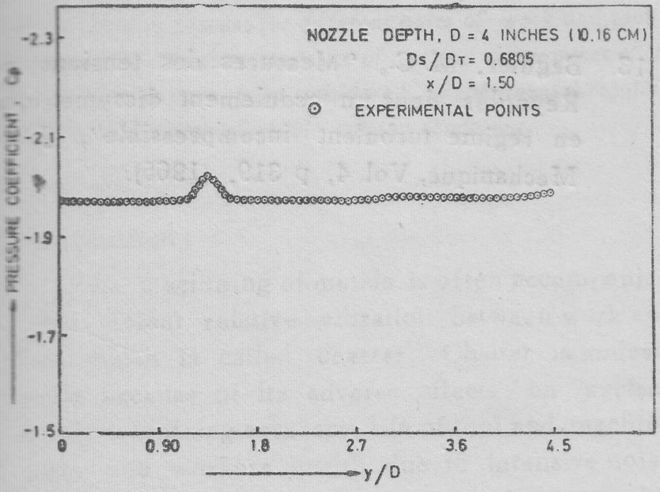

Fig. 10. Distribution of the pressure coefficient in the jet.

Conclusions :

Experimental results are obtained primarily from the data of the mixing zone of two incompressible turbulent streams and the conclusions drawn are more qualitative than quantitative.

i) The wake size close to the nozzle exit is sufficiently big to affect the flow but it gradually decreases and then disappears at an approximately same axial distance of x/D=7 for both the cases. This is quite reasonable because the boundary layer thicknesses as well as the excess mean axial velocities were approximately the same for both the cases.

ii) The velocity profile in the mixing region does not show self-preserving characteristics. This is probably due to the presence of the wake between the two streams and the large eddies and their coalescence and subsequent break down.

iii) No pressure gradient was found to exist across the streams except for a small pressure drop in the region of the wake.

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