Heat Removal Factors for Flat Plate Solar Collectors

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Abstract

New expression for heat removal factor F_R is derived. The new factors are compared with the generally accepted exponential relationship. These factors will be useful in predicting and evaluating the performances of both air and water heaters. Recommendation is made for experimental verification.

Nomenclature

A_c = Collector area, m^2.
a = surface area of collector per unit length.
a_1 = heat transfer area per unit length.
C = specific heat of heat transfer fluid KJ/KgC.
F_2, F_1 = fin efficiency of collector
F' = Collector efficiency factor.
F_1, F_1' = Heat removal factor as defined in the text.
F_R = Exponential heat removal factor.
G = fluid mass flow per unit collector area, gm/Sec.m^2.
m = fluid flow rate, g/Sec.
Q = heat collection rate. W.
q = heat collection rate, W/m^2.
r = area ratio $\frac{(a_1)}{a}$
S = net heat flux absorbed by the collector, W/m^2.
T = Variable temperature of heat transfer fluid ^0C.
T_a = ambient temperature. ^0C.
T_{pi}, T_1 = inlet temperature. ^oC.
T_{po}. To = outlet temperature. ^oC.
T_p = absorber plate temperature. ^oC.
U_f = heat transfer coefficient of the fluid, $W/m^{2o}C$.
U_L = heat loss coefficient, $W/m^{2o}C$.

Introduction

The inlet temperatures of the heat transfer fluids of collector can be ascertained easily and as such it is most convenient to evaluate the collector performance on the basis of the inlet temperature. In doing so the use of heat removal factor F_R which interrelates collector characteristics, like collector efficiency factor, fluid flow rate and the

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heat loss coefficients, is almost universal. Lunde [1] has shown that the expression F_l proposed by him is equivalent to F_R as long as the flow is high enough so that the temperature rise is less than half the potentential rise to stagnation.

The present paper is an endeavour to find new expressions for heat removal factor closer to F_R and make specific comments on the work of Lunde. This analysis is based on the following assumptions :

(1) For the same geometry of the collector the fluid heat transfer coefficient and the heat loss coefficient from the collector are invariant with the flow rates.

(2) The temperature difference between the collector plate and the heat transfer fluid is, for four different conditions considered in the paper,

i) Constant over the entire collector,

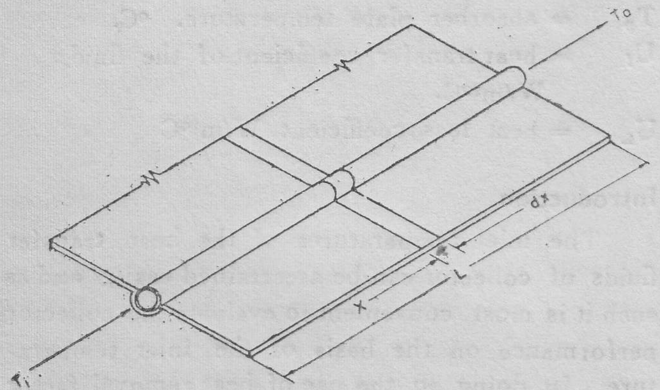
ii) Varies as linear function of length,

iii) Varies as a polynomial function of the length, and

iv) Varies exponentially along the length.

Mathematical formulation

In steady state, the performance of a solar collector as shown in figure 1 can be described



Figure—1

by an energy balance that relates the net solar energy absorbed and the thermal energy lost from the collector to the surroundings by conduction, convection and infrared radiation. The useful heat gain by the collector is then the difference between the absorbed solar radiation and the thermal loss,

$$dQ = F_2\{S - U_L(T_p - T_a)\}\, a dx \;\ldots \quad (1)$$

where $F_2 = F_1(1-r)+r$ = Average fin efficiency (2)

and r = area ratio, $\left(\frac{a_1}{a}\right)$

The mean absorber plate temperature T_p which is a function of the collector design, incident solar radiation and the entering fluid conditions, is difficult to calculate or measure [2]. Hence T_p has to be replaced by T_i, the inlet fluid temperature which is usually known.

The energy balance equation for the elementary area dx of the collector is,

$$F_2 S a\, dx = a_1 U_f (Tp - T)\, dx + F_2 a U_L (Tp - Ta)\, dx \;\ldots \quad (3)$$

or

$$Tp = \frac{F_2 sa + a_1 U_f T + F_2 a U_L Ta}{a_1 U_f + F_2 a U_L}$$

$$\therefore T_p - T = \frac{F_2\{S - U_L(T - Ta)}{rU_f + F_2 U_L} \quad (4)$$

$(Tp - T)$ is required, to find out the heat that is transferred to the fluid. In order to find out this temperature difference, four different conditions, namely (i) $(Tp - T)$ is constant, (ii) is linear (iii) is polynomial and (iv) exponential function of x, will be used.

(i) (Tp— T) constant every where.

Let us say, $(Tp - T) = (Tp_i - T_i)$ = Initial temperature difference. Heat gained by the fluid can be calculated by integrating the first term on the right of equation (3) from 0 to L,

$$\therefore Q = ra\, U_f\, (Tp_i - T_i)\, L \;\ldots\;\ldots \quad (5)$$

Puting the value of $(Tp_i - T_i)$ from (4) to (5)

$$Q = ra\,U_f - F_2 \left\{ \frac{S - U_L(Ti - Ta)}{rU_f + F_2U_L} \right\}$$

$$\text{or } Q = \frac{F_2 rU_f.Ac}{rU_f + F_2U_L}\left(S - U_L(Ti - Ta)\right)$$

$$\left[\because Ac = a\,L.\right]$$

$$\text{or } \frac{Q}{Ac} = q = \frac{1}{\frac{1}{F_2} + \frac{U_L}{rU_f}}\left(S - U_L(Ti - Ta)\right)$$

$$\therefore Q = F'\{S - U_L(Ti - Ta)\}$$

where, $F' = \frac{F_2\, rU_f}{rU_f + F_2U_L}$ = Collector efficiency factor ... ... (6)

In this particular case heat removal factor = Collector efficiency factor. Hence, the heat removal factor is independent of flow.

ii) $(T_p - T)$ as linear functisn of x

Let, $T_p - T = A + Bx$ ... ... (7)

The constants A and B are to be determined from the boundary conditons, at $x = 0$, $T_p - T = T_{pi} - Ti$ and at $x = L$, $T_p - T = Tp_o - T_o$

$$\therefore A = T_{pi} - T_i \text{ and } B = \frac{(T_{po} - T_o) - (T_{pi} - T_i)}{L}$$

$$\therefore Tp - T = (Tp_i - T_i) + \frac{(Tp_o - T_o) - (Tp_i - T_i)}{L}\,x \qquad \text{... ... (8)}$$

$$\text{Hence, } Q = raU_f\left[T_{pi} - T_i)\,L + \frac{\{(Tp_o - T_o) - (Tp_i - T_i)\}\,L}{2}\right]$$

$$= raU_f\left[Tp_o - T_o)\frac{L}{2} + (Tp_i - T_i)\frac{L}{2}\right]$$

$$= \frac{raU_fL}{2}\left[\frac{F_2(S - U_L(T_o - T_a))}{rU_f + F_2U_L} + \frac{F_2\{S - U_L(T_i - T_a)\}}{rU_f + F_2U_L}\right]$$

$$= \frac{rU_fAc}{2}\left[\frac{F_2\{S - U_L(T_o - T_i) + (T_i - Ta)\}}{rU_f + F_2U_L} + \frac{F_2\{S - U_L(Ti - T_a)\}}{rU_f + F_2U_L}\right]$$

$$\text{or } \frac{Q}{Ac}\left(1 + \frac{F_2rU_fAcU_L}{2(rU_f + F_2U_L)m_c}\right) = \left(S - U_L(Ti - T_a)\right) \times \frac{rU_fF_2}{rU_f + F_2U_L}$$

$$\text{or } Q\left(\frac{rU_f + F_2U_L}{F_2rU_f} + \frac{U_L}{2G_c}\right) = \left(S - U_L(Ti - T_a)\right)$$

$\therefore q = Fi\{S - UL(Ti - T_a)\}$ ... ... (9)

$$\text{where, } Fi = \frac{1}{\frac{rU_f + F_2U_L}{F_2rU_f} + \frac{U_L}{2G_c}}$$

$$\text{or } F_i = \frac{1}{\frac{1}{F'} + \frac{U_L}{2G_e}} \qquad \text{... ... ... (10)}$$

iii) $(Tp - T)$ as a polynomial of x

Let, $Tp - T = A_1 + B_1\,x + Cx^2$ ... ... (11)

In addition to the boundary conditions at (ii) above let us assume the third condition that $\frac{d}{dx}(Tp - T) = Q$ at $x = p^L$ where p is to be determined experimentally.

$$\therefore A_1 = (Tpi - T_i),\ B_1 = \frac{2p\{(Tp_o - T_o) - (Tpi - Ti)\}}{L\,(2p - 1)}$$

$$\text{and } C = \frac{\{(Tpi - Ti) - Tp_o - T_o)\}}{L^2\,(2p - 1)}$$

$$\therefore Tp - T = (Tpi - Ti) + \frac{2p\{(Tp_o - T_o) - (Tpi - T_i)\}}{L(2p - 1)}\,x + \frac{\{(Tpi - Ti) - (Tp_o - To)\}\,x^2}{L^2(2p - 1)} \qquad \text{... (12)}$$

By integration from O to L

$$Q = raU_f\left[(T_{pi} - T^i)\ L + \frac{p(Tpi - Ti)\}L}{(2p - 1)} + \frac{\{(Tpi - Ti) - (Tp_o - T_o)\}L}{3(2p - 1)}\right]$$

$$\text{or } Q = \frac{raU_fL}{3(2p - 1)}\left[(Tpi - Ti)\ (3p - 2) + (Tp_o - T_o)\,(3p - 1)\right]$$

Putting the values of (Tpi—Ti), (Tpo—To) and replacing $(T_o - T_i)$ by $\frac{Q}{m_c}$,

$$\frac{Q}{Ac}\left[1+3\frac{(3p-1)\,F_2 rU_f U_L}{3(2p-1)(rU_f+F_2U_L)}\right]=\frac{F_2 rU_f}{rU_f+F_2U_L}\left(S-U_L(Ti-T_a)\right)$$

$$\text{or } q = \frac{1}{\frac{rU_f+F_2U_L}{F_2 rU_f}+\frac{(3p-1)U_L}{3(2p-1)G_c}}\{S-U_L(T_i-T_a)\}$$

$$\text{or } q = F'_1\{S-U_L(Ti-T_a)\} \quad \ldots \quad \ldots \quad (13)$$

$$\text{where, } F_1 = \frac{1}{\frac{1}{F'}+\frac{(3p-1)U_L}{3(2p-1)G_c}} \quad \ldots \quad \ldots \quad (14)$$

(iv) $(T_p - T)$ as an exponential function of x

$$\text{Let } Tp - T = A\, e^{-Bx} \quad \ldots \quad \ldots \quad (15)$$

at x = 0, from equation (4)

$$Tpi - Ti = \frac{F_2}{rU_f+F_2U_L}\{S-U_L(T_i-T_a)\}$$

$$\therefore A = \frac{F_2}{rU_f+F_2U_L}\{S-U_L(T_i-T_a)\}, \text{ at } x = L$$

$$T_{po}-T_o=\frac{F_2}{rU_f+F_2U_L}\{SU_L(T_o-T_a)\}=\frac{F_2}{rU_f+F_2U_L}\{S-U_L(T_i-T_a)\}\,e^{-BL}$$

$$\text{or } S-U_L(T_o-T_a)=\{S-U_L(T_i-T_a)\}\,e^{-BL}$$

$$\text{or } \{S-U_L(T_i-T_a)-U_L(T_o-T_i)\} = \{S-U_L(T_i-T_a)\}\,e^{-BL}$$

$$\text{or } (T_o-T_i)=\frac{1}{U_L}\{S-U_L(T_i-T_a)\}(e^{-BL}-1) \quad (16)$$

$$\text{Again, } Q = m_c(T_o-T_i) = \frac{raU_fA}{B}(e^{-BL}-1)$$

$$\text{or } (To-Ti)=\frac{F_2 rU_f a\,[S-U_L(Ti-Ta)]}{mc\,(rU_f+F_2U_L)\,B}(e^{-BL}-1) \quad \ldots \quad (17)$$

from (16) and (17)

$$\frac{F_2 rU_f a\,[S-U_L(Ti-T_a)](e^{-BL}-1)]}{mc\,(rU_f+F_2U_L)\,B} = \frac{1}{U_L}\left[S-U_L(Ti-T_a)\right](e^{-BL}-1)$$

$$\text{or } \frac{F'U_L}{Gc.L} = B \quad \ldots \quad \ldots \quad \ldots \quad 18$$

$$\therefore T_p - T = \frac{F_2}{rU_f+F_2U_L}\{S-U_L(Ti-T_a)\}\times e^{-\frac{F'U_L x}{Gc.L}} \quad \ldots \quad \ldots \quad \ldots \quad 19$$

$$\text{Hence, } Q = \frac{F_2 raU_f}{rU_f+F_2U_L}\left[S-U_L(Ti-T_a)\right]\times e^{-\frac{F'U_L x}{Gc.L}}$$

on integration from 0 to L

$$Q = \frac{Ac\,Gc}{U_L}\left(1-e^{-\frac{F'U_L}{Gc}}\right)\left[S-U_L(Ti-T_a)\right]$$

$$\text{or } \frac{Q}{Ac} = q = F_R\left[S-U_L(Ti-T_a)\right] \quad \ldots \quad (20)$$

$$\text{where. } F_R = \frac{Gc}{U_L}\left(1-e^{-\frac{F'U_L}{Gc}}\right) \quad \ldots \quad \ldots \quad (21)$$

This is the universal heat removal factor.

(V) Flow Factor

In order to represent heat removal factor graphically it is convenient to take the ratio of F_R to F' and the new factor thus obtained is called the flow factor which is function of a single variable $\frac{Gc}{F'U_L}$, the dimensionless heat capacitance rate.

Dividing equation (10), (14) and (21) by F',

$$\frac{Fi}{F'} = \frac{1}{1+\frac{F'U_L}{2Gc}} \quad \ldots \quad \ldots \quad \ldots \quad (22)$$

$$\frac{Fi'}{F'} = \frac{1}{1+\frac{(3p-1)F'U_L}{3(2p-1)Gc}} \quad \ldots \quad \ldots \quad (23)$$

$$F_R = \frac{Gc}{F'U_L}\left(1-e^{-\frac{F'U_L}{Gc}}\right) \quad \ldots \quad \ldots \quad (24)$$

The variations of flow factors against $\frac{Gc}{F'U_L}$ as defined by equations (22), (23) and (24) are shown in figure 2.

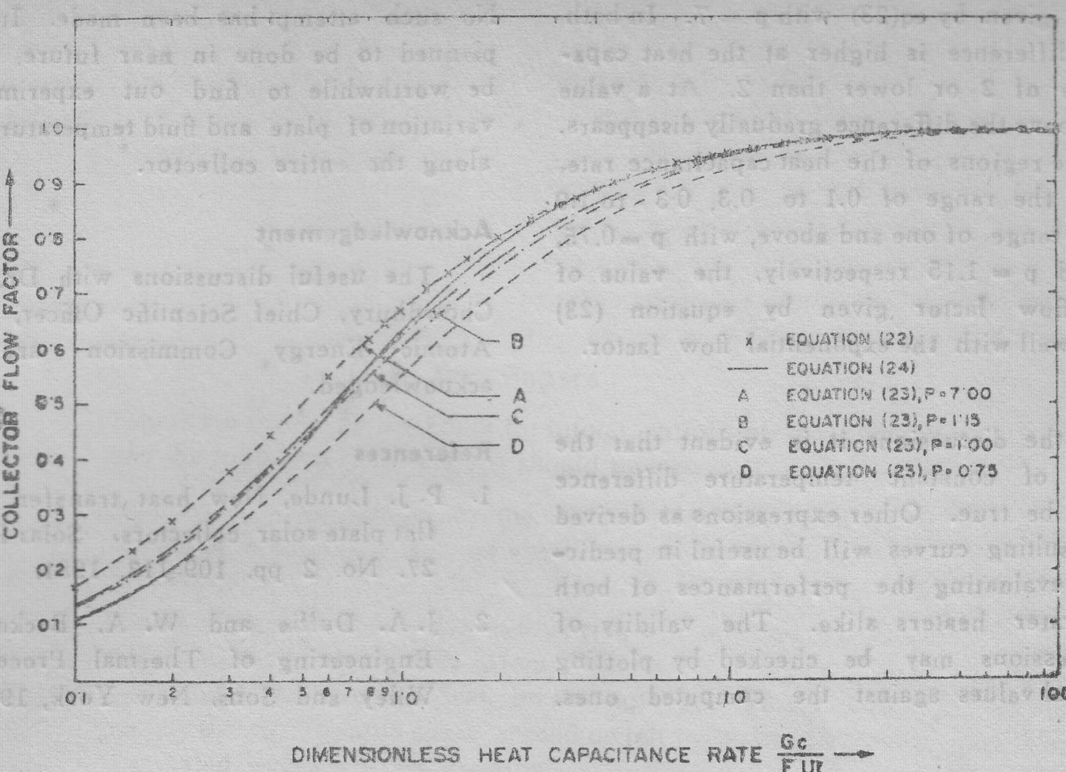


Figure – 2

Discussions

By applying relatively simple analysis, heat removal factors have been derived for a plate to fluid temperature difference as (i) constant, (ii) linear, (iii) quadratic and (iv) exponential function of the length of the collector and are found to be different. The expression for condition (ii) above appears to be same as that claimed to have been derived on the basis of constant temperature difference [I]. In the present analysis, from equation (4) it will appear that if the difference of temperature is constant, the fluid temperature must be constant. Again, if the difference and the fluid temperature are constants, the average plate temperature must also be constant. In Lunde's work, however, apart from some confusion on units, equation (7) [I] for example, the average plate temperature has been put as a linear function of area which is, in turn, a linear function of x. Hence, on integration the result of equation (10) [I] has come out exactly the same as that of linear variation of temperature difference. It may also be stated that all heat removal factors including F_R and excluding that for constant temperature difference approach to F′ as G approaches to infinity. In reference [2] also, it is stated that as the flow rate becomes very large, the temperature rise from inlet to outlet decreases towards zero but the temperature of the absorbing plate will still be higher than the fluid temperature. This difference is accounted for by the collector efficiency factor F′ and F_R can never exceed F′. Hence, the remark of Lunde that F_R becomes indeterminate as the flow approaches infinity appears to be confusing.

From figure 2 it is evident that the flow factor defined by equation (23) with p = 1.15 is the nearest to the exponential flow factor (eq. 24) while the values of flow factor according to linear variation (eq. 22) coincide with those given by eq(23) with p = 7. In both cases the difference is higher at the heat capacitance rate of 2 or lower than 2. At a value of 2 and above the difference gradually disappears. In the three regions of the heat capacitance rate, namely in the range of 0.1 to 0.3, 0·3 to 1.0 and in the range of one and above, with p = 0.75, p = 1.0 and p = 1.15 respectively, the value of collector flow factor given by equation (23) compares well with the exponential flow factor.

Conclusion

From the discussions it is evident that the assumption of constant temperature difference can never be true. Other expressions as derived and the resulting curves will be useful in predicting and evaluating the performances of both air and water heater alike. The validity of these expressions may be checked by plotting experimental values against the computed ones. No such attempt has been made. It is however, planned to be done in near future. It will also be worthwhile to find out experimentally the variation of plate and fluid temperature difference along the entire collector.

Acknowledgement

The useful discussions with Dr. S. M. M. R. Chowdhury, Chief Scientific Officer, Bangladesh Atomic Energy Commission are gratefully acknowledged.

References

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