Variation of Instability Strain with Strain Hardening Index

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Abstract

The effect of strain hardening index on instability strain has been studied previously for uniaxial and biaxial tests. But the basis of correlation with predicted values was not correct for some cases since either anisotropy was not considered in many theoretical analyses or the same values of normal anisotropy ond strain hardening index were not used for comparison. In the present analysis effects of n and R on instability strain have been analyzed by using Yamada and Yokouchi's theory. Another incremental theory has been modified to obtain the theoretical results for Aluminium Killed Steel, Soft Aluminium and Soft Brass for which experimental values were obtained by using a 254 mm (10 in) dia.diaphragm.

Notation

σ	Stress
ε	Strain
n	Strain hardening index
K	Constant in the empirical equation, $\sigma = K\varepsilon^n$
R	Average strain ratio (measure of anisotropy)

$$R = \frac{1}{4}(R_0 + 2R_{45} + R_{90})$$

R_0, R_{45}, R_{90} strain ratios along 0,45 and 90 degrees to the rolling direction.

ε_u instability strain.

Introduction

Many manufacturing processes involve deformation of a blank through the plastic range to obtain a permanent shape of a product. The deformation process might take place at a single station or at multiple stations. To reduce the manufacturing cost and time it is essential to get the deformed product in one step without involving multiple processes. But there is a limit to the deformation process at a single step and is dependent on many material factors, tool and process variables. If deformation is continued the blank will deform locally (neck down) at a certain section and for certain materials the pressure or load decreases until fracture occurs. The point of maximum load and beginning of localized deformation i.e. the point of instability is one important phenomena that need to be considered. The strain at instability has been investigated experimentally and theoretically for different cases e.g. in the case of drawing a cup with a flat headed punch theoretical analyses (1-5) have been put forward to predict the limiting drawing ratio (L. D. R.). The effect of anisotropy on LDR was also studied(1).

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The steess strain relationship in the plastic range has been approximated by some empirical equations like that of Swift, Prager, Ludwik. For most materials Ludwik's equation, $\sigma = K\varepsilon^n$ (1) has been used successfully.

Theory

The experimental stress strain curve for many materials have been used to determine the instability strain. For simple tension tests (6) it has been shown that

$$\varepsilon_u = n \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

Despite the difficulty in measuring the strains and indirect calculation diaphragm tests are preferred in many cases due to the following reasons :

1) These permit tests over a wide range of plastic strain.
2) There is no friction between the specimen and apparatus.
3) The time of preparing the specimens is less and precision machining is not essential.
4) Stress strain curve can be produced along the normal direction to the surface with the assumption of volume constancy and absence of Bauschinger effect.

Brown and Sachs (7) studied the instability of diaphragms by graphical methods but the effects of anisotropy and strain hardening exponent on instability were not considered. Hill (8) deduced one expression to determine the instability strain in terms of the strain hardening exponent as follows :

$$\varepsilon_u = \frac{4}{11}\,(2n+1) \quad \ldots \quad \ldots \quad (3)$$

But he did not consider the anisotropy of the metal. Chakrabarty and Alexander (9) used Tresca's yield criterion and associated flow rule to derive the polar strain at instability as follows.

$$\varepsilon_u = \frac{2(2-n)\,(1+2n)}{11-4n} \quad \ldots \quad \ldots \quad (4)$$

Swift (10) obtained one expression for the prediction of instability strain and Ludwik's type of relation the polar strain at instability can be obtained from the following relation.

$$\varepsilon_u = \frac{1}{2}\left[\frac{33}{10} + n - \left(\frac{729}{100} - \frac{3}{5}\,n = n^2\right)^{\frac{1}{2}}\right] \quad (5)$$

Wang and Shammamy (11) considered the anisotropy of sheet metals and used total strain and incremental strain theories in their analysis. They also described one procedure based on incremental strain theory to determine numerically and graphically the instability strain at pole for diaphragms. Their results for R = 1 together with the results of Hill, Swift, Chakrabarty and Alexander are shown in Figure 1. Wang and Shammamy also compared their instability strain

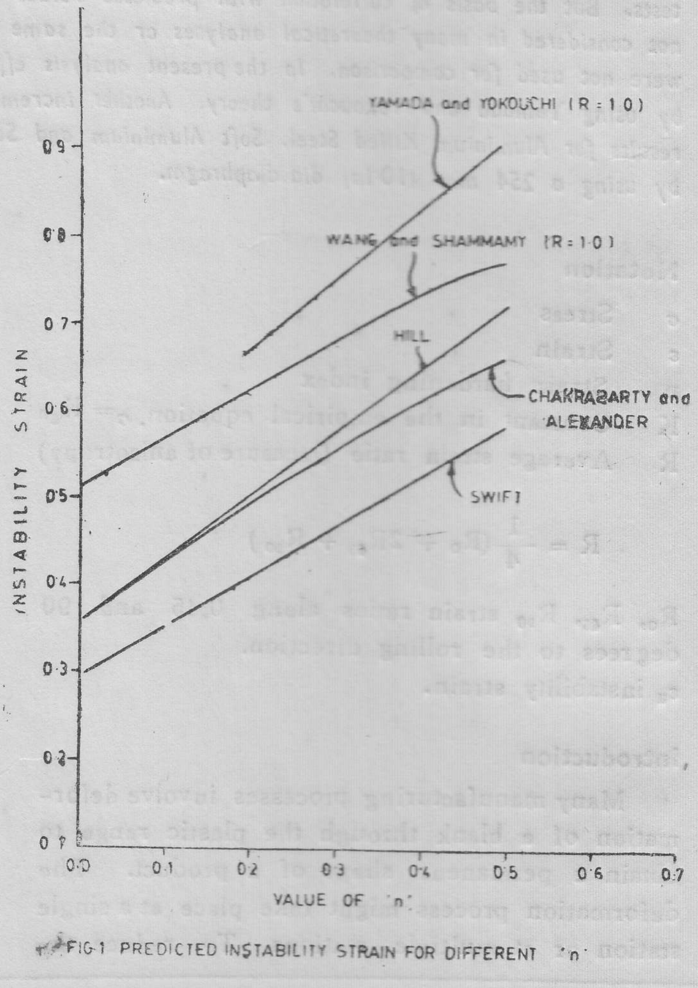

FIG-1 PREDICTED INSTABILITY STRAIN FOR DIFFERENT 'n' values with the experimental values obtained by Bramley and Mellor (12) for aluminium killed steel (R = 1. 4, n = 0.246, K = 82,000 p. s. i.) only and concluded that the polar thickness strain is overestimated by about 20 per cent. Yamada and Yokouchi (13) analysed the diaphragm problem using incremental strain theory. In the analysis it was assumed that the sheet metal was anisotropic in the normal direction. Woo (14) also used incremental strain theory for the analysis of axisymmetric forming processes but he used a total strain theory for the numerical solution due to computing difficulties. Anisotropy of the metal was not considered in this analysis. Johnson and Mellor (6) used the stress strain relationship suggested by Swift and obtained some theoretical instability strains for diaphragms and compared with the experimental value. In this analysis the anisotropy of the sheet metal was not considered.

Present work and Results

For the present work three materials (with R values below and above 1) i.e. Aluminium Killed Steel, Soft Aluminium, Soft 70/30 Brass have been selected to generate experimental values. The material properties are shown in Table : 1. Anisotropy of sheet metals should be considered since it is a rule rather than exception and arises due to different forming processes in the mill.

Material	Average Blank Thick-ness (mm)	R_0, R_{45}, R_{90}	R	$\sigma = K\varepsilon^n$	
				K(psi)	n
Aluminium Killed steel	1.133	1.78, 1.42, 1,85	1.62	71771	0.22
Soft Aluminium	0.88	0.56, 0.63, 0.73	0.64	19293	0.27
Soft 70/30 Brass	0.955	0.82, 0.9, 0 83	0 86	109240	0.51

Table 1 : Material Properties

The theory of Yamada and Yokouchi includes simple boundary condition i.e. the circumferential strain at edge is zero for all stages. The details and the difficulties of the numerical solution are given by Ilahi (15). Using this theory the effect of anisotropy and strain hardening on pole characteristics have been evaluated by Ilahi (16,17).

To analyze the effect of strain hardening on instability strain numerical calculations have been carried out for different values of n. The values of the material constants were changed to see the effect of normal anisotropy as well on instability strain. The strain at which the pressure starts to fall was taken as the instability strain. Results from the calculations are shown in Figure 2. In figure 1 the curve for R = 1 is shown for comparison with other curves.

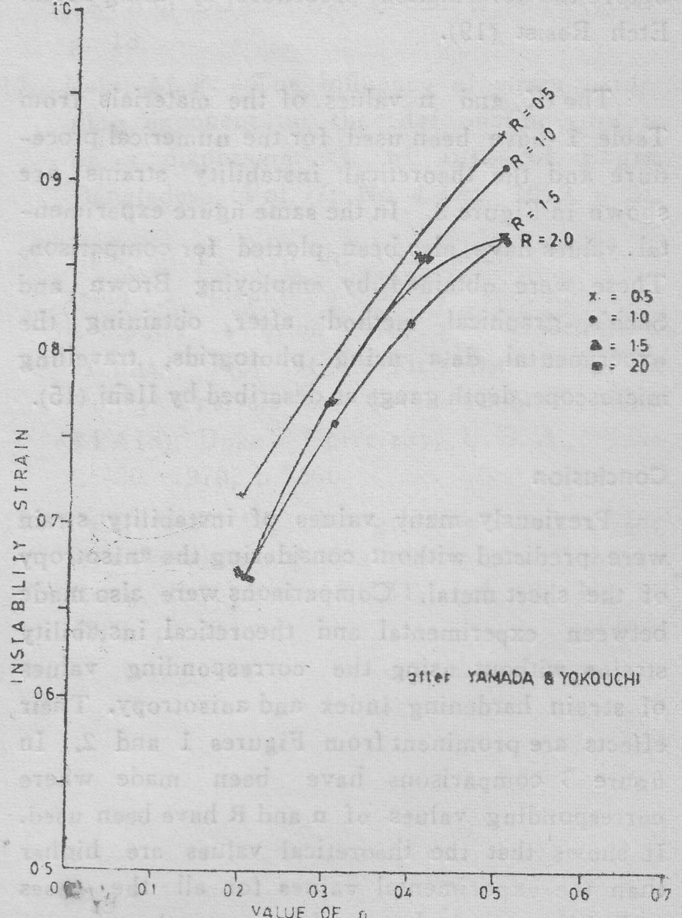

FIG 2 EFFECT OF n and R' ON INSTABILITY STRAIN

In the present analysis Woo's theory as modified by Ilahi (18) has been used to determine the instability strain. The incremental strain theory has been used throughout the numerical procedure of solution for the three materials which have been used for the experiment with a 254 mm (10 in) dia die. Previously such large diaphargm was not used and the effect of edge condition is less when thickness to diameter ratio is small. The diaphragms were deformed by unilateral pressure using oil until fracture or instability occurred and at each step of the deformation procedure the thickness distributions, heights of the poles, shapes of the diaphragms and current distances of specified elements from the centres of the diaphragms were recorded. To facilitate taking of the readings concentric circular photogrids were printed on the specimens before the deformation procedure by using Metal Etch Resist (19).

The K and n values of the materials from Table 1 have been used for the numerical procedure and the theoretical instability strains are shown in Figure 3. In the same figure experimental values have also been plotted for comparison. These were obtained by employing Brown and Sach's graphical method after obtaining the experimental data using photogrids, travelling microscope, depth gauge as described by Ilahi (15).

Conclusion

Previously many values of instability strain were predicted without considering the anisotropy of the sheet metal. Comparisons were also made between experimental and theoretical instability strains without using the corresponding values of strain hardening index and anisotropy. Their effects are prominent from Figures 1 and 2. In figure 3 comparisons have been made where corresponding values of n and R have been used. It shows that the theoretical values are higher than the experimental values for all the cases and difference is largest for brass whose strain hardening index is high. For Aluminium Killed Steel and Soft Aluminium the maximum difference is about 16 percent. The increase in instability strain due to increase in strain hardening index is reaffirmed.

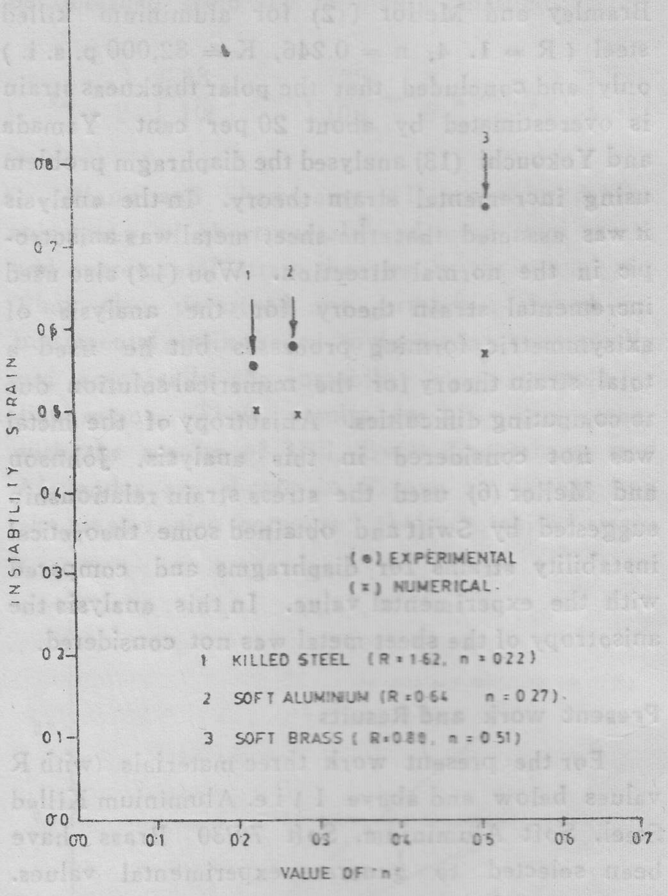

FIG 3 COMPARISON BETWEEN EXPERIMENTAL AND NUMERICAL RESULTS

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References

1. Moor. G. G. and Wallace, J, P. "The effect of anisotropy on instability in sheet metal forming." J. Inst. of Metals, Vol. 93, (1964), P. 33.

2. Ikeshima, T. Okamoto, T. and Fukuda, M, Colloquium of the I.D.D.R.G., London, (1964).

3. Chiang, D. C., Kobayashi, S. Tans. ASME, J. Eng for Industry, Paper No. 66, Prod-3 (1966).

4. Budiansky, B. and Wang N. M. J. Mech. & Phys of Solids, Vol. 14 (1966) p. 357.

5. Chakrabarty, J. and Mellor, P.B. "A new approach for predicting the limiting drawing ratio." L. Metallurgia, Italiana. No. 8 (1968) p. 791.

6. Johnson, W and Mellor P. B. 'Engineering Plasticity', Van Nostrand, London, (1973).

7. Brown, W. F. and Sachs, G. "Strength and failure cheracteristics of thin circular membranes. "Trans ASME, Vol. 70 (1948), p. 241.

8. Hill, R. "A theory of the plastic bulging of a metal diaphragm by lateral pressure." Proc. Royal Soc. Ser. 7, Vol. 41 (1950) no. 522, p. 1133.

9. Chakrabarty, J. and Alexander, J. M. 'Hydrostatic bulging of circular diaphragms." Journal of Strain Analysis, Vol. 3 (1970) No. 3, p. 155.

10. Swift, H. W. "Plastic instability under plane stress.' J. of Mech. and Phys of Solids, vol. 1 (1952) p. 1.

11. Wang, N. M. and Shammamy, M. R. "On the plastic bulging of a circular diaphragm by hydrostatic pressure." J. of Mech. and Phys of Solids, vol. 17 (1969) p. 43.

12. Bramley, A. N. and Mellor, P. B. "Some strain rate and anisotropy effects in the stretch forming of steel sheets." Int. J. of Machine Tool Design & Research, Vol. 5, (1965), p. 43.

13. Yamada, Y and Yokouchj, Y. "Analysis of the hydrostatic bulge test by the incremental theory of plasticity." (in Japanese), Seisan Kenkyu, vol. 19 No 12 (1967) p. 366, Inst. of Ind. Science. Univ. of Tokyo.

14. Woo, D. M. "The analysis of axisymmetric forming of sheet metal and the hydrostatic bulging process." Int. J. Mech. Sci., vol. 6, (1964), p. 303.

15. Ilahi, M. F. "Plastic deformation of circular metal diaphragms.' Ph.D. Thesis, University of Bradford, 1977.

16. Ilahi, M. F. "Effect of through thickness strength on bulging process.' Mech. Engg. Res. Bull, BUET, Dacca, Vol. 1. No. 1 (1978) p. 13.

17. Ilahi, M. F. "The influence of strain hardening exponent on the deformation process of a diaphragm", J. of Inst. of Engrs, Bangladesh, Vol. 7; No. 4. Oct. 1979, p. 21.

18. Ilahi, M. F. "On the correlation of results from a diaphragm theory adaptable for computer solution with the experimental results." Proc. of the First Int. Symp. on Policy Analysis and Information Systems. (PAIS), Duke University, U. S. A., June 26–30, 1979, p. 560.

19. Ilahi, M. F. "A photogridding method for strain analysis during plastic flow." Mech. Engg. Res. Bull., BUET., vol. 2, No. 1, 1979, p. 23