Radiation Heat Transfer with Relative Movement between Source and Observer

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Abstract :

Planck's quantum theory of thermal radiation was sucessfully used for radiation exchange between two surfaces under static and steady state conditions. This article shows that radiation exchange between two surfaces, which are moving with respect to each other, depends on the relative velocity between them. As a result, we cannot use the conventional relations of Thermal Radiation. This article shows that for these cases Wien's Law and Planck's Law need modifications, and Stefan-Boltzmnn constant no longer remains a constant. The application of these formulations is shown for three configurations of radiation exchange.

Notations and Nomenclature :

A = Area of radiation transfer, (m^2)

T = Absolute Temperature, (°K)

E = Total emmissive power, (ergs/cm²/sec)

F12 = Angle factor

v = Relative velocity between source and observer, (cms/sec)

c = Velocity of light, (3×10^{10} cm/sec)

u = Energy density, (ergs/cm³)

e = Energy of a quanta, [$e = h\nu$], (ergs)

k = Boltzmann constant, (1.38042×10^{-16} ergs/deg)

h = Planck's constant, (6.62517×10^{-27} ergs. sec)

q'' = Heat rate, (ergs/cm²/sec)

q' = Heat flux, (ergs/cm²)

q = Total heat flow, (ergs)

ε = Emmissivity,

ρ = Reflectivity,

τ = Transmissivity,

ν = Frequency,

λ = Wave-length, (micron)

S = Stefan-Boltzmann constant, (5.6610823×10^{-5} ergs/cm²/sec/deg⁴)

Subscripts :

c = Classical,

r = Relativistic,

λ = Monochromatic,

b = Blackbody,

1,2, = Different surfaces,

History of Thermal Radiation :

The study of thermal radiation dates back to around 1879, when Stefan gave an empirical relation connecting the total emmissive power with the absolute temperature of the body. His relation was

$$E_b = \sigma T^4 \tag{1}$$

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In 1884, Boltzmann derived eqn.(1) from thermodynamic point of view. So eqn.(1) is called Stefan Boltzmann Law. In 1893 Wien assumed radiation to be a thermodynamical engine. In 1896 he formulated the expression for monochromatic energy density distribution as

$$u_b\lambda = \frac{C_1}{\lambda^5} e^{-\frac{C_2}{\lambda T}} \qquad (2)$$

where C_1 and C_2 are constants. This expression was found to fit experimental curves, satisfactorily in the shorter zone of wave-length. But in the longer zone of wave-length, the experimental values were higher than the formulated values.

In 1900 Lord Rayliegh and James Jeans took a different approach to thermal radiation. Their expression for energy density distribution was

$$u_b\lambda = \frac{8\pi kT}{\lambda^4} \qquad (3)$$

Eqn.(3) agreed well with the experimental values in the longer zone of wave length. But it was quite interesting to note that the formulated values from eqn.(3) were too large than the experimental values in the shorter zone of wave-length.

After the failure of Wiens Law and Rayliegh-Jeans Law it become a real puzzle to the physicists to determine the real nature of thermal radiation. The problem was solved by the turn of this century, when Max Planck gave the revolutionary idea of quantum machanics in 1901.

Planck's Law :

Planck postulated that energy emitted from a source is not continuous as was thought in the electromagnatic theory of thermal radiation. He said that energy is emitted in discrete quantities, called 'quanta'. Energy content in each quanta is

$$e = h\nu \qquad (4)$$

where 'h' is a new universal constant called 'Planck's Constant'. He therefore, concluded that since energy emmission is not continuous, we cannot integrate to find out the spectral distribution, as was done by Rayliegh and Jeans. Planck's formulation for energy density distribution is

$$u_b\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\left(e^{\frac{hc}{k\lambda T}} - 1\right)} \qquad (5)$$

The graphical representation of eqn. (2), (3) and (5) is shown in Fig. (1). To find out the intensity of radiation from the density of radiation

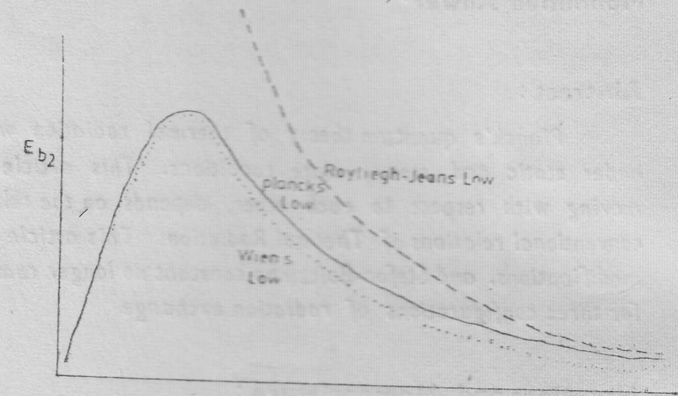


Fig-1. Spectral distribution of Thermal Radiation

λ

$$E_b\lambda = \frac{c}{4} u_b\lambda \qquad (6)$$

With the help of eqn.(5) and (6) we obtain

$$E_b\lambda = \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{\left(e^{\frac{hc}{k\lambda T}} - 1\right)} \qquad (7)$$

To find out the maximum intensity of monochromatic emmissive power, $dE_b\lambda/d\lambda = 0$.

With this we obtain from eqn.(7)

$$\lambda_m T = \frac{1}{4.965} \cdot \frac{ch}{k} \qquad (8)$$

where λ_m is the wave-length of maximum intensity of monochromatic emmissive power. Eqn.(8) follows Wien's Law, since c, h and k are constants.

To find out the total emmissive power E_b, we integrate eqn(5) over the whole length of electromagnetic spectrum.

$$E_b = \int_0^\infty E_{b\lambda} \, . \, d\lambda$$

$$= \int_0^\infty \frac{2\pi c^2 h}{\lambda^5} \cdot \frac{1}{\left(e^{\frac{hc}{k\lambda T}} - 1 \right)} \, d\lambda$$

$$= \left(\frac{2\pi^5 k^4}{15 c^2 h^3} \right) T^4 \qquad (9)$$

This is simply Stefan-Boltzmann Law, with Stefan-Boltzmann constant

$$\sigma = \frac{2\pi^5 k^4}{15 c^2 h^3} \qquad (10)$$

Interested readers can get a full treatment of the theory of thermal radiation from Kaplan (1), Born (2), Jacob (3), Rajam (4), Rydnik (5), Ponomarev(6).

Doppler's Effect :

When there is a relative motion between a source and an observer, the frequency of the waves send by the source differs from that recieved by the observer. This is true for all types of spherical waves. This effect is called Doppler's effect.

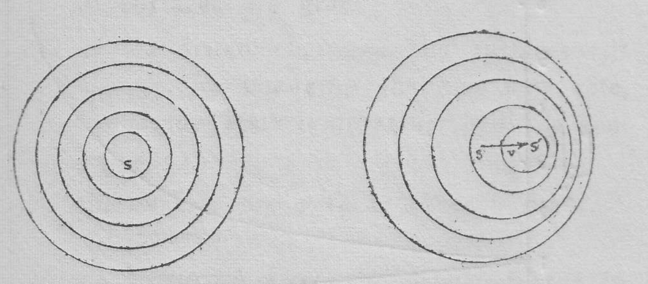


Fig-2 Dopplers effect is the compression of waves on one side and rarefaction on the other side.

Consider Fig.—2, in which a source of spherical waves S is moving with a velocity v. Considering a time t, let the number of waves send by S be equal to n. Therefore the wave-length

$$\lambda = \frac{ct}{n} \qquad (11)$$

But in this time t, the source has moved a distance $x = vt$. Let us further suppose that the source is moving towards the observer, (We consider this situation to be having a positive relative velocity). Therefore the apparent wave-length received by the observer

$$\lambda_c = \frac{ct - vt}{n} \qquad (12)$$

From eqn. (11) and (12), we get

$$\frac{\lambda_c}{\lambda} = (1 - \frac{v}{c})$$

$$= 1 - \beta \text{ where } \beta = v/c \qquad (13)$$

will be positive if source is moving towards the observer, and its value will be negative if the source is moving away from the observer. Therefore the altered frequencies are given by

$$\frac{v}{v_c} = 1 - \beta \qquad (14)$$

If the observer is in motion, and source is stationary, the altered frequency is given by

$$\frac{v_c}{v} = 1 + \beta \qquad (15)$$

The treatment of Doppler's effect given above is true for mechanical waves only, such as sound waves. These waves need a medium to go through. So there is a physical difference betw een observer in motion, source stationary and source in motion, observer stationary. But for the cases of electro-magnetic waves, the situation is absolutely different. From the first postulate of relativity, however, all internal reference frames are equivalent, and from the second postulate of relativity, velocity of light is invariant, that is it is independent of velocity of source or that of the observer. Thus in the case of electromagnetic waves, Doppler's effect should depend only on the relative velocity between the source and the observer. In this case, in addition to the altera-tion of frequency by Doppler's effect, there will also be some alteration due to relativity.

Thus to obtain the alteration in the frequency we have to take relativistic approach. We then arrive at the expression

$$\frac{v_r}{v} = \frac{1+\beta}{\sqrt{1-\beta^2}}$$

Therefore, the altered wave-length is

$$\frac{\lambda_r}{\lambda} = \frac{\sqrt{1-\beta^2}}{1+\beta} \qquad (16)$$

For further readings readers can see Rossi(7), or any other standard book.

Bergmann (8) mentions that Ives* measured the relativistics Doppler's effect. Ives measured the change of $H\beta$ lines emitted by hydrogen 'canal rays' using accelerated voltage upto 18,000 volts. Velocity attained by him was 1.8×10^8 cms/sec, So $\beta = 0.006$. He confirmed the relativistic transformation given by eqn.(16).

Formulation :

So far mentioned here, on this basis we will see, how the expressions for radiation exchange changes for moving cases. Thermal radiation is emitted as heat quanta. So these have length. Wave-length in their cases is defined as the distance between two consecutive discrete quanta. Thus these will also be subjected to Dopplers effect.

Radiation which is emitted by the source is dependent on the thermodynamical properties of the emitting source. Under any circumstance, as long as the thermodynamical properties of the source remains constant, there will be no change in the radiation emmission by the source. When source and the receiver are stationary, the wave-length of radiation received by the receiver is equal to the wave-length emitted by the source. But in our case, the emitter emits the radiation of a wave-length of λ, but the receiver receives the radiation at an altered wave-length of λ_r. This situation can be considered similar to that the receiver is receiving the radiation from a stationary source which is emitting radiation at the value of wave-length of λ_r. So the radiation that is received by the observer, $E_b\lambda_r$, can be calculated by replacing λ for λ_r from eqn.(16). Thus

$$E_b\lambda_r = \frac{2\pi c^2 h}{p^5 \lambda^5} \cdot \frac{1}{\left(e^{\frac{hc}{kp\lambda T}} - 1\right)} \qquad (17)$$

$$\text{where } p = \frac{\sqrt{1-\beta^2}}{1+\beta}$$

Total emmissive power

$$E_b\lambda = \int_0^\infty E_b\lambda' \, d\lambda$$

$$= \int_0^\infty \frac{2\pi c^2 h}{p^5\lambda^5} \frac{1}{\left(e^{\frac{h_c}{kp\lambda T}} - 1\right)} d\lambda$$

$$= \frac{2\pi^5 k^4}{15c^2h^3p} T^4$$

$$= F(v). T^4 \qquad (18)$$

We see that Stefan-Boltzmann Law changes, and a 'velocity function' F(v), takes the place of Stefan-Boltzmann constant. The variation of p with β is shown in Fig. (3). To obtain the wave-

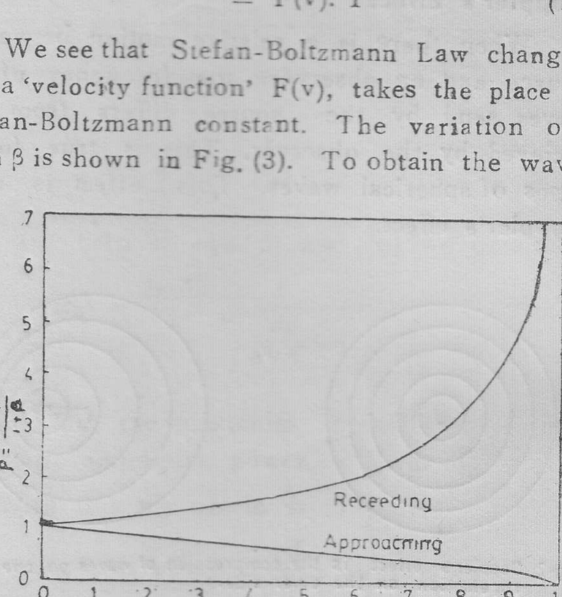


Fig-3. Variation of p with β

*H. E. Ives, Journal of American Optical Society, No. 28, p—215, (1938) length of maximum monochromatic emmissive power, we take the same earlier approach with eqn. (17), and obtain

$$\lambda_m T = \frac{1}{4.965} \cdot \frac{ch}{pk} \text{cm.}^\circ\text{K} \quad (19)$$

Hence these show that Wien's Displacement Law also needs a modification when applied for moving cases.

We have applied the theory of thermal radiation to static bodies. Bodies exchanging thermal radiation can be grouped among any of the following three configurations :

i) two surfaces could not 'see' themselves,
ii) one surface could 'see', the other one could not 'see' itself,
iii) both the surfaces 'could 'see' themselves,

The author is restricting the present analyses to case (i) only for the following three cases of radiation exchange ;

a) Between a differential area and a rectangle plate,
b) Between two coaxially placed circular plates,
c) Between two equal, coaxial rectangular plates,

Before going into detailed analyses of each case, the author wants to make the following simplifying assumptions :

i) all surfaces are gray,
ii) as the distance between the surfaces will change, by changing the heat flow rate, the steady state temperature will be kept constant,
iii) the surface are parallel to each other.

Shape factor is a very important factor in radiation exchange between two bodies separated by distance. If the two bodies form an enclosure, it is easier to find out the angle factor from one surface to another. Simple geometric structures can also be solved easily. Angle factor depends upon the dimension of the radiating surface, and upon the distance between them. In the present case, the distance between the two radiating surface is changing, so angle factor will also change continuously. Jacob (9) gives the expressions for angle factors for some geometric configurations. The relations given by him are adopted from different sources. The angle factor relations given below are adopted from Jacob (9), and are given here in a modified form.

a) Two surfaces, one of differential, and another of rectangular area,

$$F_{12} = \frac{1}{2\pi}\left[\frac{B}{\sqrt{C^2+B^2}} \operatorname{Sin}^{-1} \frac{1}{\sqrt{1+C^2+B^2}} + \frac{1}{\sqrt{1+C^2}} \operatorname{Sin}^{-1} \frac{B}{\sqrt{1+C^2+B^2}}\right] \quad (20)$$

This configuration can be used for the following configurations only.

$$F_{12} = F_{1-I} + F_{1-II} + F_{1-III} + F_{1-IV}$$

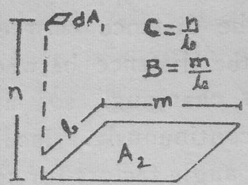
b) Two coaxially placed circular plates.

$$F_{12} = \frac{(1+C^2+B^2)+\sqrt{(C^2+B^2+1)^2-4B^2}}{2B^2} \quad (21)$$

c) Two equal coaxial rectangular plates.

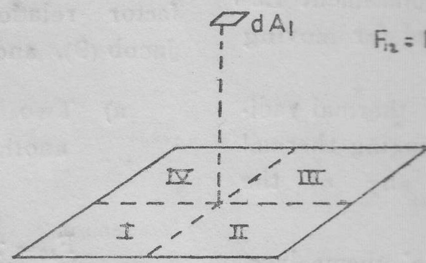
$$F_{12} = \frac{1}{\pi}\left[\frac{C^2}{B} \text{ in } \frac{C^4+C^2B^2+C^2+B^2}{C^4+C^2B^2+C^2} - \frac{2C}{B}\tan^{-1}\frac{1}{C} + 2\sqrt{1+C^2}\tan^{-1}\frac{B}{\sqrt{1+C^2}} \; 2C\tan^{-1}\frac{B}{C} + \frac{12}{B}\sqrt{C^2+B^2}\tan^{-1}\frac{1}{\sqrt{C^2+B^2}}\right] \quad (22)$$

Equation (20) and (22) are represented graphically in Fig. 6 and Fig. 7 with B as a parameter. The values of angle factors are calcutated by Computer, and it was observed that for a fixed value of C, eqn. 21 does not change appreciably with the change in B.



$$F_{12} = \frac{1}{2\pi}\left[\frac{B}{\sqrt{C^2+B^2}}\sin^{-1}\frac{1}{\sqrt{1+C^2+B^2}} + \frac{1}{\sqrt{1+C^2}}\sin^{-1}\frac{B}{\sqrt{1+C^2+B^2}}\right] \quad (21)$$

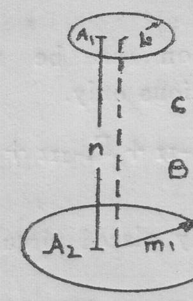
This configuration can be used for the following case only



$$F_{12} = F_{1-I} + F_{1-II} + F_{1-III} + F_{1-IV}$$

Fig. 4

(b) Two coaxially placed circular plates



$$C = \frac{n}{b} \qquad B = \frac{m}{b}$$

$$F_{12} = \frac{(1+C^2+B^2) - \sqrt{(C^2+B^2+1)^2 - 4B^2}}{2B^2} \quad (22)$$

Radiation exchange between two closely spaced infinite plate is given by ;

$$q'' = \frac{\sigma\,(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1} \quad (23)$$

In this case, since the plates are closely spaced, and are assumed to be infinite in dimension. The angle factor is assumed to be 1. But we cannot make this assumption in our case, as it is clear from fig. 6 and fig. 7 that in most cases the angle factor is much less than 1.

To find out the net radiation exchange between two surfaces in our cases, consider Fig. 5

Fig 5 Radiation transfer between two plates with $F_{12} \neq 1$

Radiation loss from surface 1 is given by :-

$$q' = E_1A_1 - E_1A_1(F_{12}\rho_2F_{21}\alpha_1 + F_{12}\rho_2F_{21}\rho_1F_{12}\rho_2F_{21}\alpha_1 + \ ) - A_2E_2(F_{21}\alpha_1 + F_{21}\rho_1F_{12}\rho_2F_{21}\alpha_1 + F_{21}\rho_1F_{12}\rho_2F_{21}\rho_1F_{12}\rho_2F_{21}\alpha_1 + \dots)$$

where $E = \epsilon E_b$

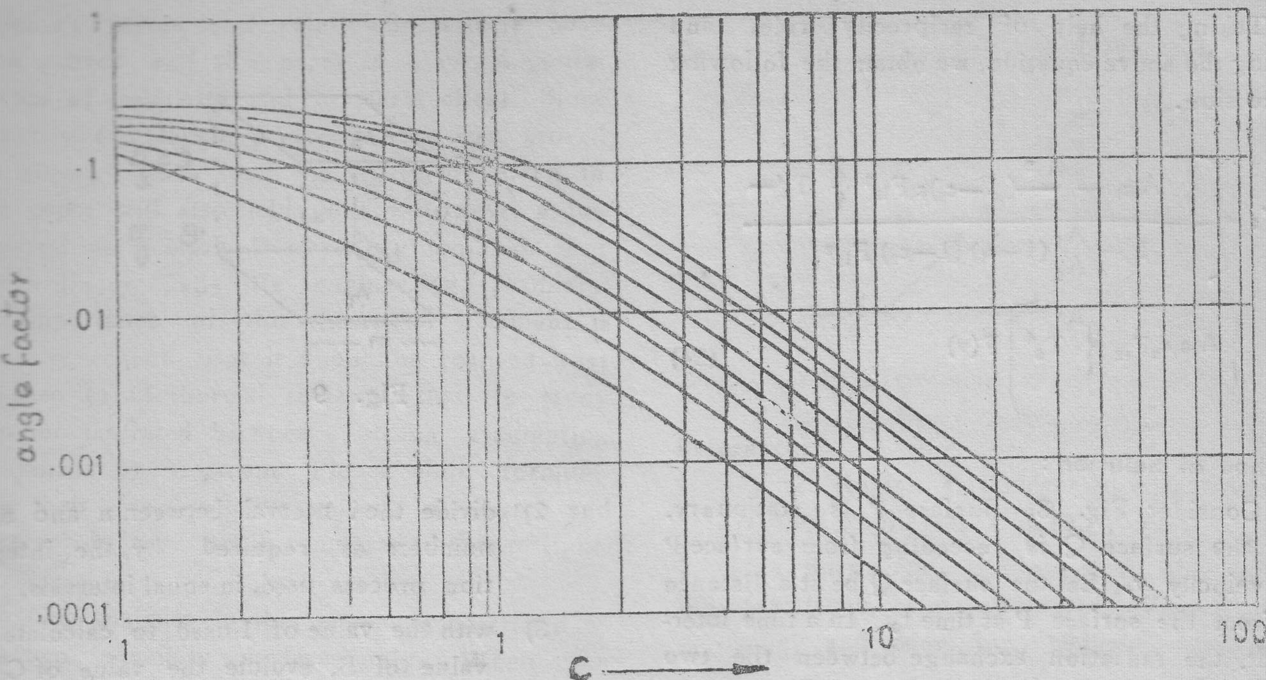


Fig-6· Vanation of angle factor with distance for a differential area and a rectangular area.

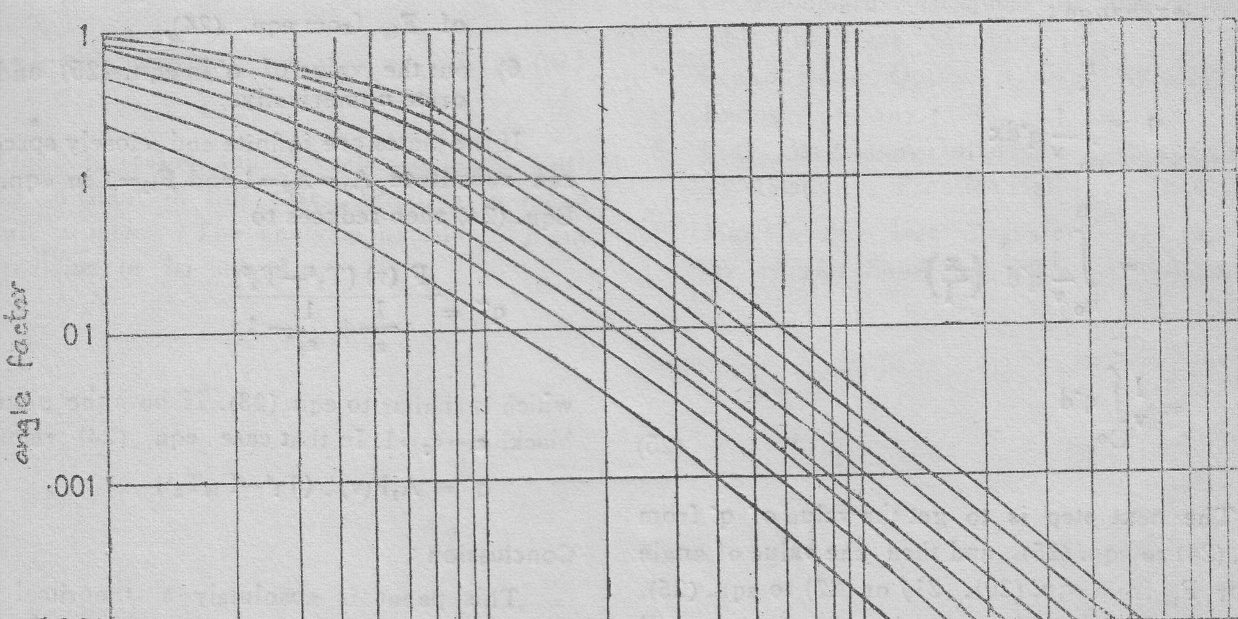


Fig-7. Variatation of angle factor with distance for two equal and coaxial rectangular plates.

Taking the help of reciprocity rule, and solving the above equation, we obtain the following expression.

$$q'_1 = \left[\frac{\left\{ A_1\varepsilon_1 - \frac{A_1^2}{A_2}(1-\varepsilon_2)\varepsilon_1 F_{12}^2 \right\} T_1^4 - \left\{ A_1\varepsilon_1\varepsilon_2 F_{12} \right\} T_2^4}{1 - \frac{A_1}{A_2}(1-\varepsilon_1)(1-\varepsilon_2) F_{12}^2} \right] F(v) \qquad (24)$$

Method of Solution :

Consider Fig. 8. Surface P is stationary, and the surface Q is receeding from surface P at a velocity v. Let the surface Q be at a distance x_0 from the surface P at time t_0. In a time interval dt, the radiation exchange between the two surface is :

$$dq = q'.dt$$

Now, $dt = (1/v).dx$

Therefore the total thermal radiation exchange :

$$q = \int_{n_0}^{n} \frac{1}{v} q' dx$$

$$= \int_{n_0}^{n} \frac{1}{v} q' d\left(\frac{x}{l}\right)$$

$$= \frac{1}{v}\int_{C_0}^{C} q' d \qquad (25)$$

The next step is to get the value of q′ from eqn. (24) to eqn. (25). and then the value of angle factor F_{12} from eqn. (20), (21) or (22) to eqn. (25). It is obvious that the resulting expression will not be very nice one to integrate. So the only feasible way of solution is to integrate the final expression of eqn. 25 numerically. To do it

1) find B. This should be always less than 1,

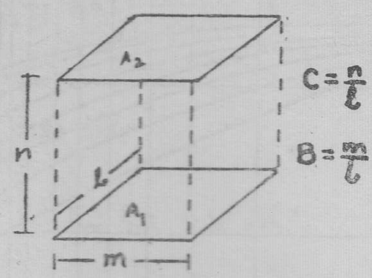


Fig. 9

2) divide the interval between n and n_0, in numbers as required by the integration process used, in equal intervals,

3) with the value of l used to calculate the value of B, evalute the value of C for each interval of step (2),

4) find out the value of F_{12} either from the curves or from the formula,

5) find out the value of q for each value of F_{12} from eqn. (24),

6) put the value of q′ in eqn, (25) and integrate numerically.

If the plates are infinite and closely spaced, we can substitute $A_1 = A_2 = 1$ and $F_{12} = 1$ in eqn. (24). Eqn. (24) then reduces to

$$q'' = \frac{F(v)\,(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

which is similar to eqn. (23). If both the plates are black, $\varepsilon_1 = \varepsilon_2 = 1$. In that case eqn. (24) reduces to

$$q' = A_1 F(v) . (T_1^4 - F_{12} T_2^4)$$

Conclussion :

This paper is absolutely a theoritcal treatment of radiation exchange between source and sink which are moving with respect to each other. The author agrees that the modifications he has mentioned here are marked only at high velocities. But the fact remains that

Doppler's effect and relativistic effects have been proved and this paper is a direct consequence of relativity and Doppler's effect. Since relativity. and Doppler's effects have been proved the author thinks that the treatment given in this paper will also hold good. Howevr, experimental verification is obviously required. For our ordinary daily life engineering problems the importance of this theoritical treatment is basically absent. But it should be realised that the theories of thermal radiation that we study are not saturated because a strong assumption was made by Wien and Planck while formulating their laws—relative velocity if source and observer is very small compared to that of light.

The analyses in this paper is restricted to the two coaxially surfaces only. When the surfaces are not coaxial, Dopplers shift has a different form than that given in eqn. (16). The general expression for Doppler's shift is given by :

$$\frac{\lambda_r}{\lambda}=\frac{\sqrt{1-\beta^2}}{1+\beta \cos \alpha} \qquad (27)$$

This expression shows even Doppler's shift is not constant in this case. So it is a more difficult problem. The analysis of this problem still remains to be solved.

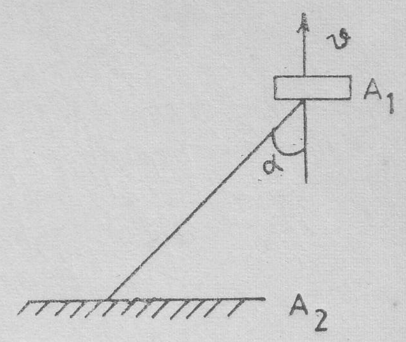


Fig-9.

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