An Approximate Boundary Layer Solution

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Abstract: An unsteady boundary layer subjected to an unsteady pressure gradient for a two-dimensional flow is considered initially. The corresponding non-linear boundary layer equations are then converted to a single ordinary differential equation of a scale function by an approximate integral method proposed by Bianchini et.al[1]. As case studies, three different steady cases are taken into account to validate the solution procedure adopted here. Closed form solutions of these three cases are obtained and compared with known results.

Keywords: *Approximate Solution, Boundary Layer*

INTRODUCTION

A complete calculation of the boundary layer for a given body with the aid of the Prandtl's boundary layer equations is, in many cases, so cumbersome and time-consuming that it can only be carried out with the aid of computer. It is, therefore, desirable to pose at least approximate methods of solutions, to be applied in cases when an exact solution of the boundary layer equations can not be obtained with a reasonable amount of work, even if their accuracy is only limited. Von Karman[2] was the first to devise an approximate method and considered a steady flow taking the potential flow simply as a function of longitudinal distance x. The details of this integral method are found in Schlichting[3].

On the otherhand a number of important fluid-dynamic phenomena are governed by boundary-layer unsteadiness. Two of these phenomena can be cited as stall flutter and rotating stall. Because of the presence of an unsteady pressure field the behaviour of the boundary-layer are still not well understood, though some solutions are available for special cases[4,5,6]. In view of the importance of the unsteady boundary-layer flows, Bianchini et al. introduced a more simpler approximate method, with the help of the powerful error function, to calculate the characteristics of the unsteady boundary-layer flows, taking the potential flow simply as a function of time. Later this approximate method was further developed by Socio and Pozzi[7] with a rigorous mathematical approach taking the potential flow to be a function of 'x' and 't'.

The essence of the first approximate method developed by Bianchini et al. and Socio and Pozzi is to assume a similarity solution even in those situations where similarity solutions do not exist, and to find a suitable scale factor for the similarity variable. In comparison with the classical series and numerical solutions and other approximate methods which also heavily rely on numerical computations, the above method does not require lengthy calculations. Moreover, no linearization is required. A successful application of the first approximate method has recently been done by Palekar and Sarma[8] to the case of a steady boundary-layer flow with suction and blowing. Very recently Sattar[9] modified the method of Bianchini et al. by introducing a time-dependent length scale along with the scale

function for the similarity variable considered by Bianchini et al. The first order differential equation that Bianchini et al. obtained for the scale function was obtained by Sattar in a much more simpler form which lead to a simple solution. In this research work our aim is to extend the work of Sattar by taking the potential flow to be a function of '*x*' and '*t*'. Our aim would thus be to attain a similarity solution by adopting a scale function for the similarity variable to be a function of '*x*' and '*t*' as proposed by Bianchini et al.
As in the work of Sattar along with the scale function we will also introduce a time-dependent length scale which has the bearing with the boundary-layer thickness.

THE GOVERNING EQUATIONS AND THE METHOD OF SOLUTIONS

The basic two-dimensional equations governing the unsteady incompressible laminar boundary layer flow past a flat plate are given by (Schlichting[3])

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

with the boundary conditions

$$\left.\begin{array}{l} u=v=0 \ \text{ for } \ y=0 \\ u=U(x,t) \text{ as } \ y\to\infty \end{array}\right\} \qquad (3)$$

where *x* and *y* are the Cartesian co-ordinates along the flow and normal to it respectively, '*t*' is the time, *u* and *v* are the velocity components along *x* and *y* directions and *U*(*x*, *t*) is the velocity of the potential flow far away from the boundary, p is the pressure, ρ is the density of the fluid and ν is the Kinematic coefficient of viscosity.
The potential flow *U*(*x*, *t*) is considered to be known, it determines the pressure distribution within the flow as shown below.
We know that at the outer edge of the boundary layer the parallel component of velocity '*u*' becomes equal to that in the outer flow, *U* (*x*, *t*). Since there is no large velocity gradient here the viscous term in the original Navier-Stokes equations vanish for large values of Reynolds number, and consequently, *U*(*x*, *t*) satisfies the following Navier-Stokes equation for a two-dimensional incompressible flow.

$$\frac{\partial u'}{\partial t}+u'\frac{\partial u'}{\partial x}+v'\frac{\partial u'}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad (4)$$

where u' and v' are the parallel velocity components of a general non viscous flow.

Thus satisfying *U*(*x*, *t*) for the above equation (4), for the outer flow we obtain

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}. \qquad (5)$$

Now using (5) in equation (2), the basic governing equations (1) and (2) and the boundary conditions (3) turn out respectively to be

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{6}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2} \tag{7}$$

$$\left.\begin{array}{l} u=v=0 \ \text{for} \ y=0 \\ u=U(x,t) \ \text{as} \ y\to\infty \end{array}\right\} \tag{8}$$

Since our main objective is to obtain a similarity solution based on a scale function, we therefore, consider a similarity variable η as

$$\eta=\frac{Y}{h(X)} \tag{9}$$

where $Y=\dfrac{y}{\sigma}, X=\dfrac{x}{\sigma}$ and $\sigma=\sigma(t)$ is a length scale but a function of time. σ is therefore considered to be a characteristic length and $h(X)$ is a scale function. As mentioned in the introduction the length scale σ has a bearing with the boundary layer thickness which can be ascertained later.

With the aid of the above similarity variable we now assume a dimensionless velocity profile as

$$u=U(x,t)f(\eta) \tag{10}$$

The continuity equation (6) now may be written as

$$v=-\int\frac{\partial u}{\partial x}dy. \tag{11}$$

Introducing (9), (10) and (11) in equation (7) we obtain

$$f\frac{\partial U}{\partial t}+Uf'\frac{\eta}{\sigma}\frac{d\sigma}{dt}\left(\frac{X}{h}\frac{dh}{dX}-1\right)+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{dU}{dt}+U\frac{dU}{dx}+\nu\frac{\partial^2 u}{\partial y^2}. \tag{12}$$

We now integrate the above equation (12) across the boundary layer from zero to infinity with respect to y, to obtain.

$$\int_0^\infty f\frac{dU}{dt}dy+\int_0^\infty Uf'\frac{\eta}{\sigma}\frac{d\sigma}{dt}\left(\frac{X}{h}\frac{dh}{dX}-1\right)dy+\int_0^\infty\left(u\frac{\partial u}{\partial x}+v\frac{du}{dy}\right)dy$$

$$=\int_0^\infty\frac{dU}{dt}dy+\int_0^\infty U\frac{dU}{dx}dy+\int_0^\infty\nu\frac{\partial^2 u}{\partial y^2}dy \tag{13}$$

which finally reduces to the following form

$$\frac{\sigma^2}{\nu}\frac{1}{U}\frac{dU}{dt}h\alpha_1+\frac{h\sigma}{\nu}\frac{dU}{dX}(\alpha_1+2\alpha_3)-\frac{\sigma}{\nu}\frac{d\sigma}{dt}\left(X\frac{dh}{dX}-h\right)\alpha_2+\frac{\sigma U}{\nu}\frac{dh}{dX}\alpha_3=\frac{\alpha_4}{h} \tag{14}$$

$$\left.\begin{aligned} &\text{where } \alpha_1 = \int_0^\infty (1-f)\,d\eta, \alpha_2 = \int_0^\infty \eta f' d\eta \\ &\alpha_3 = \int_0^\infty f(1-f)\,d\eta \text{ and } \alpha_4 = \left(\frac{\partial f}{\partial \eta}\right)_{\eta=0} \end{aligned}\right\} \quad (15)$$

(15)

The potential flow U(*x*, *t*) is now considered to be separable and can be written in the form

$$U(x,t) = V(t)F(X). \quad (16)$$

Hence using (16) in (14) we obtain

$$\frac{\sigma^2}{\nu}\frac{1}{V}\frac{dV}{dt}h\alpha_1 - \frac{\sigma}{\nu}\frac{d\sigma}{dt}\frac{X}{F}\frac{dF}{dX}h\alpha_1 - \frac{\sigma}{\nu}\frac{d\sigma}{dt}\left(X\frac{dh}{dX} - h\right)\alpha_2 + \frac{h\sigma}{\nu}V\frac{dF}{dX}\alpha_5 + \frac{U\sigma}{\nu}\frac{dh}{dX}\alpha_3 = \frac{\alpha_4}{h} \quad (17)$$

where $\alpha_5 = \alpha_1 + 2\alpha_3$.

In order to obtain a simple form of solutions of (17) one can try a class of solution by setting,

$$\frac{\sigma^2}{\nu}\frac{1}{V}\frac{dV}{dt} = -\frac{\sigma}{\nu}\frac{d\sigma}{dt} \quad (18)$$

Therefore,

$$\frac{dV}{V} = -\frac{d\sigma}{\sigma} \quad (19)$$

which on integration yields

$$V(t) = A\sigma^{-1}$$

Let at $t=t_0$, $V=V_0$ and $\sigma=\sigma_0$.
Then from (19) we have $A=V_0\sigma_0$.
Hence the potential flow given by (16) can be put in the form

$$U(x,t) = V(t)F(X) = \frac{A}{\sigma}F(X) = \frac{V_0\sigma_0}{\sigma}F(X) = \frac{V_0}{\sigma^*}F(X) \quad (20)$$

where $\sigma^* = \dfrac{\sigma}{\sigma_0}$

We now define Reynolds number R as

$$R = \frac{U\sigma}{\nu}.$$

Hence using (19) and (20) in (17) we obtain

$$-\frac{\sigma}{\nu}\frac{d\sigma}{dt}h\alpha_1 - \frac{\sigma}{\nu}\frac{d\sigma}{dt}\frac{X}{F}\frac{dF}{dX}h\alpha_1 - \frac{\sigma}{\nu}\frac{d\sigma}{dt}\left(X\frac{dh}{dx} - h\right)\alpha_2$$

$$+\frac{R}{F}\frac{dF}{dX}h\alpha_5+R\frac{dh}{dX}\alpha_3=\frac{\alpha_4}{h}. \tag{21}$$

The two dimensional boundary layer equation (6) and (7) are now reduced to a single first order differential equation for the scale function $h(X)$ except for the term $\frac{\sigma}{\nu}\frac{d\sigma}{dt}$ where the time 't' appears explicitly. But since the notion of the present problem is to find a similarity solution, the similarity condition requires that $\frac{\sigma}{\nu}\frac{d\sigma}{dt}$ must be a constant.

Therefore let $\frac{\sigma}{\nu}\frac{d\sigma}{dt}=K$ (say) (22)

Thus integrating (22), taking into consideration the condition that $\sigma=0$ at $t=0$, we obtain $\sigma=\sqrt{2K\nu t}$ (23)

Using (22) the equation (21) reduces to

$$-Kh\alpha_1-K\frac{X}{F}\frac{dF}{dX}h\alpha_1-K\left(X\frac{dh}{dX}-h\right)\alpha_2+\frac{R}{F}\frac{dF}{dX}h\alpha_5+R\alpha_3\frac{dh}{dX}=\frac{\alpha_4}{h}$$

or, $-Kh(\alpha_1-\alpha_2)+(R\alpha_3-KX\alpha_2)\frac{dh}{dX}+(R\alpha_5-KX\alpha_1)h\frac{1}{F}\frac{dF}{dX}=\frac{\alpha_4}{h}$ (24)

An integration to the above equation thus gives the scale parameter h(X) and hence a solution to the velocity profiles. The constant of integration arising out of this integration will be determined by adding an initial condition to the set of boundary conditions adopted in (8).

GENERAL SOLUTION

In order to have a general solution for the scale parameter h(X) of the equation (24), it is now necessary to make an assumption of the function $f(\eta)$ for the velocity distribution proposed in (10).
A convenient choice for $f(\eta)$ is made to be

$$f(\eta)=erf(\eta) \tag{25}$$

With this choice of $f(\eta)$ the constants defined in (15) are then integrated out as

$$\alpha_1=\frac{1}{\sqrt{\pi}},\quad \alpha_2=\frac{1}{\sqrt{\pi}},\quad \alpha_3=\frac{\sqrt{2}-1}{\sqrt{\pi}}\ \text{and}\ \alpha_4=\frac{2}{\sqrt{\pi}}.$$

Since $\alpha_1=\alpha_2$ the equation (24) now takes the form

$$(R\alpha_3-KX\alpha_2)\frac{dh}{dX}+(R\alpha_5-KX\alpha_1)\frac{h}{F}\frac{dF}{dX}=\frac{\alpha_4}{h} \tag{26}$$

The only boundary condition needed to solve this equation is

$h(X)=0$ for $X=0$ (26a)

The above equation plays the key role of the problem considered in this research. An integration to this equation would thus give the scale parameter for different cases to be considered and hence would lead to the solutions for the velocity profiles of the respective cases. Three different steady cases, which are of practical importance, will thus be taken into account.

SOLUTIONS FOR STEADY CASES

In the steady case the characteristic length $\sigma(t)$ and the potential flow $U(X, t)$ are now considered as

$$\sigma(t) = L \text{ and } U(X, t) = U(X) \text{ where } X = \frac{x}{L}.$$

Now from (22)

$$\frac{\sigma}{\upsilon}\frac{d\sigma}{dt} = \frac{\sigma}{\upsilon}\frac{d}{dt}(L) = 0 \tag{27}$$

From equation (20) we can write

$$U(X) = U_0 F(X) \tag{28}$$

Therefore the general equation (26) reduces to the form

$$R\alpha_3\frac{dh}{dX} + R\alpha_5\frac{h}{F}\frac{dF}{dX} = \frac{\alpha_4}{h} \tag{29}$$

Here the Reynolds number $R = \dfrac{U\sigma}{\upsilon}$ reduces to $R = \dfrac{UL}{\upsilon}$.

The above equation (29) thus describes the basic scale function equation for the steady cases to be considered below.

FLOW PAST A FLAT PLATE AT ZERO INCIDENCE

In the case of a flat plate the outer potential flow is taken to be uniform and hence we have

$$F(X) = 1.$$

Thus equation (29) takes the form

$$hdh = \frac{\alpha_4}{R_0 \alpha_3} dX \tag{30}$$

$$\text{where } R_0 = \frac{U_0 L}{\upsilon}.$$

Integrating (30) and using $h(X) = 0$ for $X = 0$ we obtain

$$h(X) = \sqrt{\frac{2\alpha_4 X}{R_0 \alpha_3}} \tag{31}$$

Hence the similarity variable

$$\eta = \frac{Y}{h} = \frac{Y}{\sqrt{X}} \sqrt{\frac{R_0 \alpha_3}{2\alpha_4}} = \xi \sqrt{\frac{R_0 \alpha_3}{2\alpha_4}} \tag{32}$$

$$\text{where } \xi = \frac{Y}{\sqrt{X}}.$$

Now from (10) and (32), the velocity distribution for the flow past a flat plate is obtained as

$$\frac{u}{U_0} = erf\left\{ \xi \sqrt{\frac{R_0 \alpha_3}{2\alpha_4}} \right\} \tag{33}$$

The velocity profile obtained due to the above distribution is plotted in Figure 1. A comparison of this result is also made with those of Blasius exact solution and the experimental results of Hill and Stenning[10].

Now the shearing stress corresponding to the solution of (33) is obtained as

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = \frac{\mu}{h} \left(\frac{\partial u}{\partial \eta} \right)_{\eta=0} = \frac{\mu U_0}{Lh} \left(\frac{\partial f}{\partial \eta} \right)_{\eta=0}$$

$$= \frac{\mu U_0}{Lh} \left(\frac{2}{\sqrt{\pi}} e^{-\eta^2} \right)_{\eta=0} = \frac{2\mu U_0}{Lh\sqrt{\pi}}$$

$$= \sqrt{\frac{2\alpha_3}{\pi \alpha_4}} \left(\frac{\mu U_0}{L\sqrt{X}} \right) \sqrt{\frac{U_0 L}{\eta}}.$$

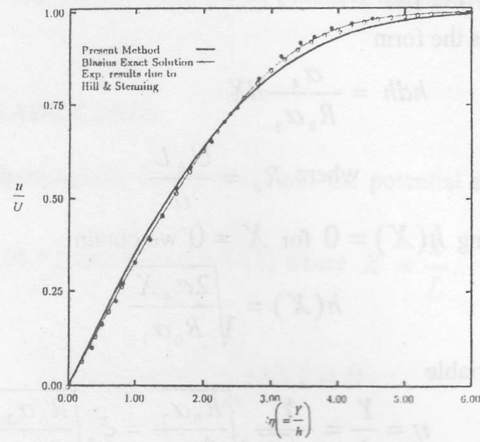

Figure 1: Steady velocity profiles for a flat plate

Hence the skin friction denoted by c_f is obtained as

$$\frac{1}{2}c_f = \frac{\tau_0}{\rho U_0^2} = \sqrt{\frac{2\alpha_3}{\pi\alpha_4}}\frac{\upsilon}{LU_0}\sqrt{\frac{U_0 L}{\upsilon X}}$$

$$= \sqrt{\frac{2\alpha_3}{\pi\alpha_4}}\sqrt{\frac{\upsilon}{LU_0 X}} = \sqrt{\frac{2\alpha_3}{\pi\alpha_4}}\sqrt{\frac{1}{\frac{U_0 X}{\upsilon}}}$$

$$= \sqrt{\frac{2\alpha_3}{\pi\alpha_4}}R_X^{-\frac{1}{2}} \left[\text{where } R_X = \frac{\upsilon}{U_0 X}\right]$$

$$= \sqrt{\frac{\sqrt{2}-1}{\pi}}R_X^{-\frac{1}{2}} = .363019\ R_X^{-\frac{1}{2}}.$$

FLOW IN A CONVERGENT CHANNEL

For a flow in a convergent channel the potential flow is taken to be

$$U(x) = -\frac{u_1}{x} = -\frac{u_1}{L}\left(\frac{1}{X}\right), \qquad u_1 > 0$$

with $u_1 > 0$, $U(x)$ represents two-dimensional motion in a convergent channel with flat walls.

In this case $R = \frac{UL}{\upsilon} = -\frac{u_1}{\upsilon X}$.

Now corresponding to the distribution (20) we have $F(X) = -\frac{1}{X}$.

Therefore, the equation (29) for this case reduces to

$$h\frac{dh}{dX} - \alpha_6\frac{h^2}{X} = -\frac{\alpha_4 \upsilon X}{u_1\alpha_3} \quad (34)$$

where $\alpha_6 = \frac{\alpha_5}{\alpha_3}$.

To find a solution of (34), let $z = h^2$,
so that the equation (34) takes the form

$$\frac{dz}{dX} - 2\alpha_6\frac{z}{X} = -\frac{2\alpha_4 \upsilon X}{u_1\alpha_3} \quad (35)$$

A solution of (35) is obtained as

$$z = h^2 = \frac{\upsilon\alpha_4}{u_1\alpha_3(\alpha_6 - 1)}X^2 \quad (36)$$

Therefore, $h = \left(\sqrt{\frac{\upsilon\alpha_4}{u_1\alpha_3(\alpha_6 - 1)}}\right)X$

and $\eta = \frac{Y}{h} = \frac{y}{x}\sqrt{\frac{u_1}{\upsilon}}\sqrt{\frac{\alpha_3(\alpha_6 - 1)}{\alpha_4}}$.

The velocity distribution for this case is obtained as

$$\frac{u}{U} = f(\eta) = erf(\eta) = erf\left\{\frac{y}{x}\sqrt{\frac{u_1}{\upsilon}}\sqrt{\frac{\alpha_3(\alpha_6 - 1)}{\alpha_4}}\right\} \quad (37)$$

The velocity profile thus obtained from (37) is plotted in Figure 2 along with exact solution due to Pohlhausen[11].

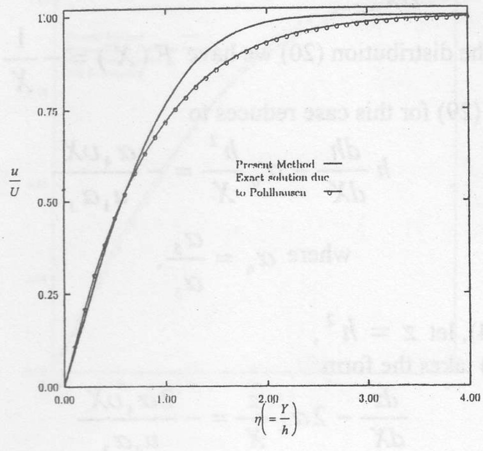

Figure 2: Velocity profiles for the flow in a convergent channel

FLOW PAST A WEDGE

For the flow past a wedge the potential flow is taken to be

$$U(X) = u_1 x^m$$
$$= u_1 (LX)^m$$
$$= u_1 L^m X^m \quad (\text{since} \quad X = \frac{x}{L}) \tag{38}$$

Where u_1 is a constant and the component m is chosen with respect to the similarity condition of Falkner and Skan equation.

$$\text{Then } R = \frac{UL}{\upsilon} = \frac{u_1 L^{m+1} X^m}{\upsilon}.$$

Thus comparing (38) with (16) we obtain

$$F(X) = X^m.$$

Therefore the equation (29) reduces to

$$h\frac{dh}{dX} + \frac{m\alpha_5}{\alpha_3}\frac{h^2}{X} = \frac{\alpha^4}{\alpha_3 u_1 L^{m+1} X^m} \tag{39}$$

As before let $z = h^2$, hence (39) takes the form

$$\frac{dz}{dX} + \alpha_7 \frac{z}{X} = \frac{2\alpha_4 \upsilon}{\alpha_3 u_1 L^{m+1} X^m} \tag{40}$$

where $\alpha_7 = \dfrac{2m\alpha_5}{\alpha_3}$.

A solution for $h(X)$ in the present case is thus obtained as

$$h = \sqrt{\frac{2\alpha_4 \upsilon}{\alpha_3 u_1 L^{m+1}(\alpha_7 + 1 - m)}}\, X^{\frac{1-m}{2}}.$$

Therefore $\eta = \dfrac{Y}{h} = y\sqrt{\dfrac{u_1}{\upsilon}}\sqrt{\dfrac{\alpha_3(1-m+\alpha_7)}{2\alpha_4}}\, x^{\frac{m-1}{2}}$.

Thus the velocity distribution for this case is obtained as

$$\frac{u}{U} = f(\eta) = erf(\eta) = erf\left\{y\sqrt{\frac{u_1}{\upsilon}}\sqrt{\frac{\alpha_3(1-m+\alpha_7)}{2\alpha_4}}\, x^{\frac{m-1}{2}}\right\}.$$

$$= erf\left(\eta_1\sqrt{\frac{\alpha_3(1-m+\alpha_7)}{2\alpha_4}}\right) \tag{41}$$

where $\eta_1 = y\sqrt{\dfrac{u_1}{\nu}}\, x^{\frac{m-1}{2}}$

Following the distribution for the flow past a wedge the velocity profiles $\left(\dfrac{u}{U}\right)$ have been plotted against $\eta_1 = y\sqrt{\dfrac{u_1}{\upsilon}}\, x^{\frac{m-1}{2}}$ for two values of $m\left(=\dfrac{1}{3} \text{ and } 1\right)$ in Figures 3 and 4 respectively.

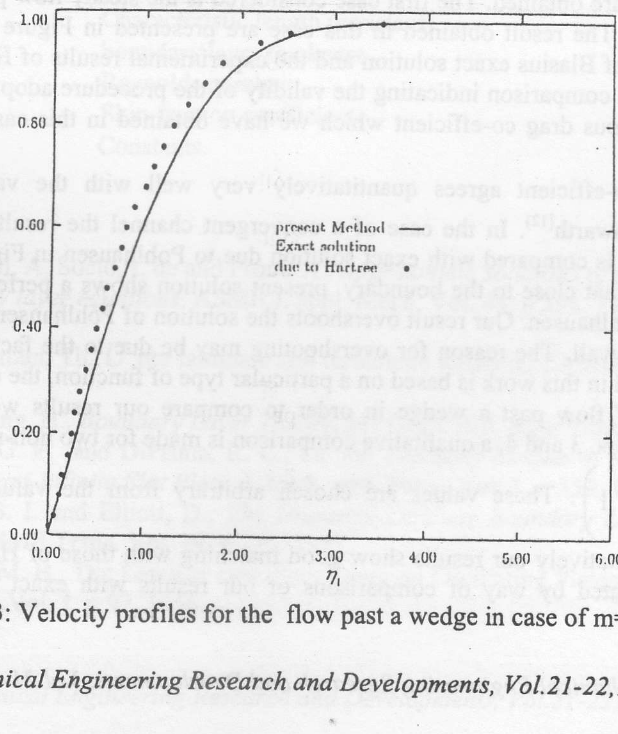

Figure 3: Velocity profiles for the flow past a wedge in case of m=1/3

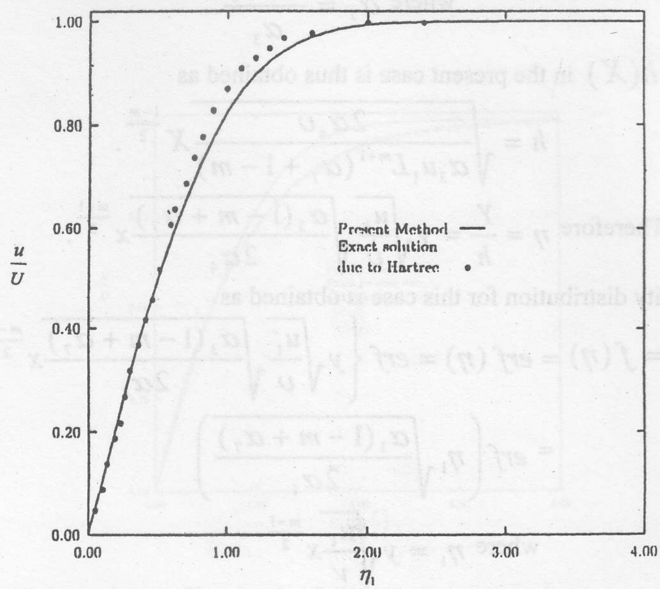

Figure 4: Velocity profiles for the flow past a wedge in case of m=1

DISCUSSIONS

Based on the unsteady boundary layer equations, as special cases, steady solutions for different flows are obtained. The first case considered is the steady flow past a flat plate at zero incidence. The result obtained in this case are presented in Figure 1 and compared with the result of Blasius exact solution and the experimental results of Hill and Stenning. It shows a good comparison indicating the validity of the procedure adopted in the present study. The viscous drag co-efficient which we have obtained in this case in the form of skin-friction co-efficient agrees quantitatively very well with the value $.33206\, R_X^{-\frac{1}{2}}$ obtained by Howarth[12]. In the case of a convergent channel the result obtained by the present method is compared with exact solution due to Pohlhausen in Figure 2. It appears from Figure 2 that close to the boundary, present solution shows a perfect matching with the result of Pohlhausen. Our result overshoots the solution of Pohlhausen in the region far away from the wall. The reason for overshooting may be due to the fact that the integral method adopted in this work is based on a particular type of function, the error function. As for the case of flow past a wedge in order to compare our results with the results of Hartree[13] in Figs. 3 and 4, a qualitative comparison is made for two non-negative values of $m\left(=\frac{1}{3} \text{ and } 1\right)$. These values are chosen arbitrary from the values considered by Hartree. Quantitatively our results show good matching with those of Hartree. It has thus been demonstrated by way of comparisons of our results with exact and experimental results that integral solution procedure adopted here is advantageous over the time consuming classical and numerical methods, as for as the simplicity is concerned.

NOMENCLATURE

X, Y	Coordinates along the plate and normal to it
u, v	Velocity components
t	Time
U	Potential velocity
ρ	Density
μ	Coefficient of viscosity
$\nu\left(=\dfrac{\mu}{\rho}\right)$	Kinematic coefficient of viscosity
σ (t)	Length scale
$X\left(=\dfrac{x}{\sigma}\right)$	Dimensionless length along x-axis
$Y\left(=\dfrac{y}{\sigma}\right)$	Dimensionless length along y-axis
$h\left(=\dfrac{Y}{X}\right)$	Scale factor
f	Function of η
F	Function of X
V	Function of t
L	Characteristic length (constant)
δ	boundary layer thickness
R	Reynolds number
f_c	Skin-friction coefficient
$\alpha_1, \alpha_2, \alpha_3$ & α_4	Constants.

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