On the Validity of Taylor's Hypothesis on Wall Turbulence

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Abstract: The validity of Taylor's hypothesis of frozen turbulence has been the issue of much debate, especially when applied to flows with strong shear and high turbulence intensities. In the past, Taylor's hypothesis was used by various researchers for the quantitative interpretation of the structure angle of the eddies on the basis of double-velocity correlations (e.g. Alving et al. [1]) or velocity-wall pressure or velocity-wall shear stress correlations (e.g. Brown & Thomas [2], Rajagopalan & Antonia [6]. In light of the ambiguity associated with Taylor's hypothesis, naturally, there are resultant uncertainities in terms of the measured structure angle. Subsequently there is a need to investigate how do these uncertainties effect the structure angle measurements and as well as to examine the validity of Taylor's hypothesis when applied to two-point double-velocity correlation measurements in an anisotropic shear flow.

Received: *24 February 1997*
Accepted: *20 November 1997*

Keywords: *Turbulence, Boundary layer, Talylor's hypothesis.*

INTRODUCTION

Taylor [8] pointed out that, if the turbulence level were low, the time variation in the velocity u observed at a fixed point in the flow would be approximately the same as those due to the convection of an unchanged spatial pattern past the point with a constant velocity U_C (now generally referred to as the *convection velocity*), i.e.

$$u[x,t] \doteq u[x-U_C t,0]$$

where x and t represent distance measured downstream in the mean flow direction and time respectively. This is classicaly known as the *Taylor's hypothesis of frozen turbulence*. In his original paper on grid turbulence, Taylor assumed the convection velocity is equal to the local mean velocity and showed that this hypothesis approximately holds true in grid turbulence.

Starting from the equations of motion, Theodorsen [9], with some intuitive arguments, proposed that a turbulent shear flow consists of *horseshoe-vortices* which are inclined in the flow-direction at an angle of 45° (on the average). Later Head and Bandyopadhyay [4] showed by very convincing flow-visualization studies with smoke that a turbulent boundary layer appears to consist of a "forest" of Hairpin vortices; these vortices can be seen in their 1979 cine film to lean in the downstream direction at approximately 45°. However, a flow-visualization experiment is always qualitative in nature. Hence one needs a quantitative approach to the problem. Intuitively one could suggest that the velocity signature at any point is likely to be correlated with that at any other point within the same eddy, whereas the velocity signatures from two different eddies are more likely to be uncorrelated. This implies that the two-point space-time correlation measurements can be used in this regard.

The normalized two point space-time cross-correlation (or double velocity correlation) between the velocities at two points A at x and B at x+r in the flow can be defined (by using a suitably defined co-ordinates system as discussed later),

$$R_{ij}[r,\tau] \equiv R_{ij}[\Delta x, \Delta z, \tau] = \frac{\overline{u_{iA}u_{jB}}}{\sqrt{\overline{u_{iA}^2}}\sqrt{\overline{u_{jB}^2}}} \qquad (1)$$

where t is the time-shift, **x** is the position vector at A and **r** is the distance between the two points defined as **r** [Δx, Δy, Δz]. Here and in all discussions to follow, it is assumed that the position of A is closer to the wall and it is chosen as the origin of Δx and Δz co-ordinates (all the measurements were taken along the centre line of the wind tunnel i.e. $y_A = y_B = 0$ implying that $\Delta y = 0$). Please note that in this paper we will present the experimental results involving the correlation of the stream-wise component u_1 of the fluctuating velocity only. Also note that all length scales will be normalized by the boundary layer thickness δ_H such that

$$\ell^* \equiv \frac{\ell}{\delta_H}$$

where ℓ is any length scale.

On the basis of our assumption that the velocities at two points with in the same eddy is correlated, one would heuristically suggest that the averaged or ensemble averaged quantity $R_{11}[\Delta x, \Delta z, 0]$ will have a maxima at $\Delta x = \Delta x_m$ for a particular Δz such that

$$\tan\theta_i = \frac{\Delta z}{\Delta x_m} \qquad (2)$$

where θ_i could be interpreted as the *inferred* or *effective* average inclination of the eddies to the streamwise direction.

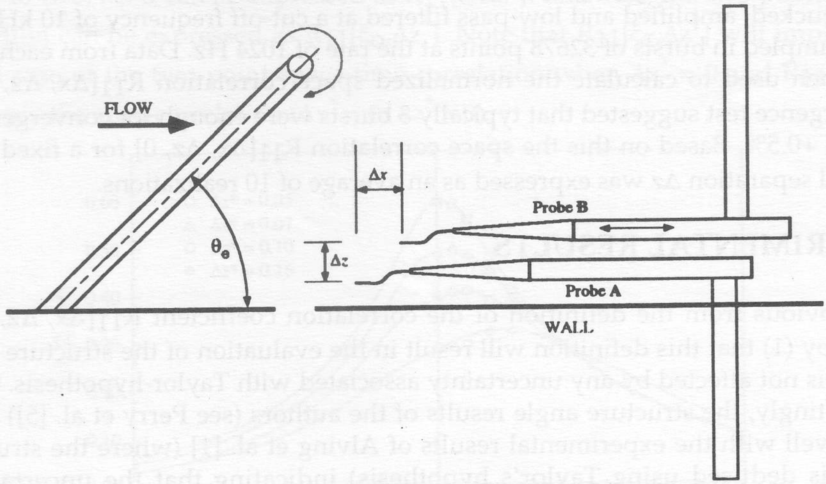

Fig. 1: Schemaic diagram (not to scale) of the probe arrangement

THE EXPERIMENTAL SET-UP AND METHODOLOGY

The wind tunnel used for this investigation was an open-return suction type of conventional design. The working section was 2500 mm long and had a cross-sectional dimensions of 613 mm x 309 mm and all measurements were taken at x = 1780. The freestream turbulence intensity in the working section was about 0.3%. The boundary layer was tripped by a tripping wire of diameter 1.2 mm.

All the hot wire measurements performed in this investigation were made using unlinearized constant-temperature hot wire anemometers designed and built at Melbourne University. The hot wire probes used were normal-wire probes designed in such a way that they offered reliable close to the wall measurements and also the interference between the probes were minimal while taking the correlation measurements. The probe arrangement used in this experiment is shown in figure 1.

Data was acquired on line to a 32-bit PC using an Analog-Devices RTI-860 data acquisition board. With 12-bit resolution, 4 channels could be sampled simultaneously at a maximum rate of 50 kHz. The operating window of the A/D board was set to ±5V implying that the resolution of the board would be ± 2.5 mV.

The measurements of streamwise space correlations $R_{11}[\Delta x, \Delta z, 0]$ were carried out at a nominal reference freestream velocity of 10 m/s which corresponded to

$R_q = 2613$ and a boundary layer thickness $\delta_H = 41$ mm at $x_A = 1780$ mm. For each streamwise position of the upper probe, the signal from both the anemometers were bucked, amplified and low-pass filtered at a cut-off frequency of 10 kHz and then sampled in bursts of 32678 points at the rate of 1024 Hz. Data from each burst was then used to calculate the normalized space correlation $R_{11}[\Delta x, \Delta z, 0]$. A convergence test suggested that typically 8 bursts were enough for convergence to within +0.5%. Based on this the space correlation $R_{11}[\Delta x, \Delta z, 0]$ for a fixed wall-normal separation Δz was expressed as an average of 10 realizations.

EXPERIMENTAL RESULTS

It is obvious from the definition of the correlation coefficient $R_{11}[\Delta x, \Delta z, 0]$ as given by (1) that this definition will result in the evaluation of the structure angle which is not affected by any uncertainty associated with Taylor-hypothesis. Quite interestingly, the structure angle results of the authors (see Perry et al. [5]) agree quite well with the experimental results of Alving et al. [1] (where the structure angle is deduced using Taylor's hypothesis) indicating that the uncertainties associated with Taylor's hypothesis do not affect the structure angle measurements. To investigate this further and to resolve the issue whether Taylor's hypothesis is still applicable for smaller Δz^*, the two-point space correlation as obtained by physically moving the probe and the two-point space time correlation as obtained using Taylor's hypothesis are shown in figure 2 for various Δz^* at $z^*_A = 0.26$. Here, for convenience, the nondimensional parameter ξ is defined as

$$\xi = \Delta x^* - \frac{\tau U_c}{\delta_H} \tag{3}$$

where t is the time shift, and U_c and Δz^* are the convection velocity and the nondimensional streamwise probe separation respectively, the subscripts 0 and t imply special cases of x corresponding to t = 0 and $\Delta x^* = 0$ respectively, i.e.,

$$\xi_o = \xi|_{\tau=0} = \Delta x^*$$

$$\xi_\tau = \xi|_{\Delta x^*=0} = \frac{-\tau U_c}{\delta_H}$$

As we are interested only in the correlation of the streamwise component of the fluctuating velocities, when the time shift parameter is transformed to a distance l (say) (using Taylor's hypothesis i.e. $l = U_c t$), then the streamwise separation Δx^* and l^* can be grouped together in the new nondimensional variable x. Introduction of this new nondimensional variable x has the implication that, now the number of variables in the argument list of the correlation coefficient will reduce to two, i.e. it can be expressed as $R_{11}[\xi,\Delta z^*]$. Likewise, the two point space correlation can be expressed as $R_{11}[\xi_o,\Delta z^*]$. Note that $R_{11}[\xi_\tau,\Delta z^*]$ will imply the special case of the two-point space-time correlation when $\Delta x^* = 0$ and $R_{11}[\xi_\tau,0]$ will imply the auto-correlation ($\Delta x^* = 0, \Delta z^* = 0$).

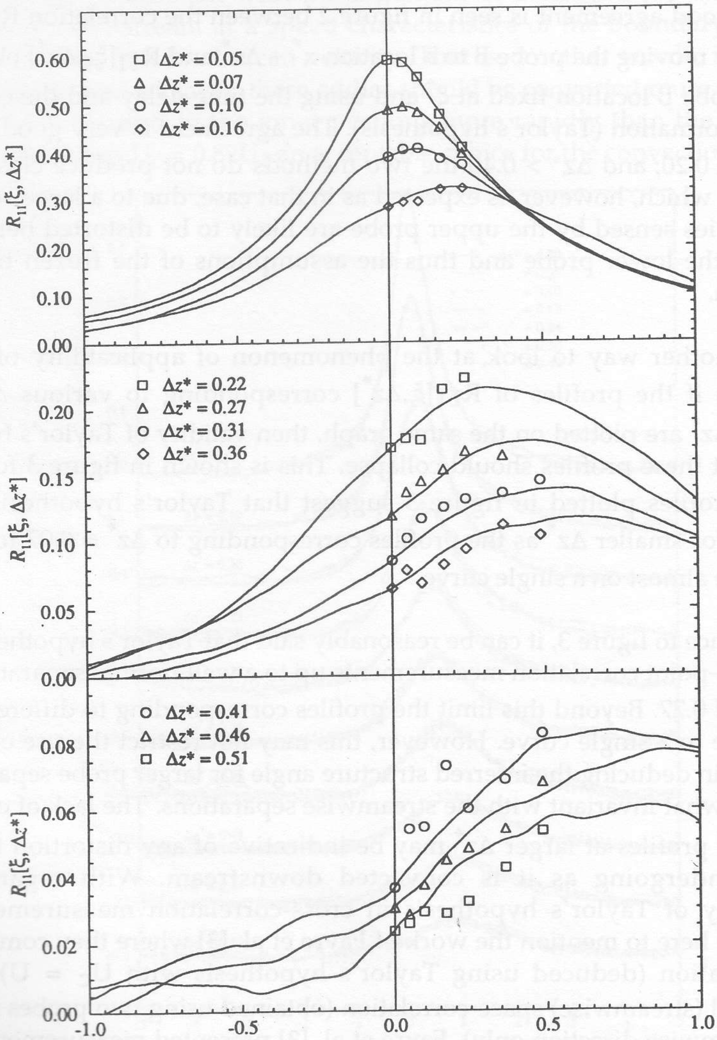

Fig. 2: Comparison between correlations obtained by actual probe movement and by using Taylor's hypothesis (for converting the time-shift to space-shift) for various Δz^* at $z^*_A = 0.26$. The solid lines represent $R_{11}[\xi_\tau, \Delta z^*]$ and the symbols represent $R_{11}[\xi_0, \Delta z^*]$. Note the change of scale for the ordinates

Using this new variable ξ, and defining ξ_m as ξ corresponding to the maxima in $R_{11}[\xi,\Delta z^*]$, the inferred average structure angle can now be found from

$$\theta_i = \arctan[\frac{\Delta z^*}{\xi_m}] \tag{4}$$

Overall, a good agreement is seen in figure 2 between the correlation $R_{11}[\xi_o,\Delta z^*]$ obtained by moving the probe B to a location $x^* + \Delta x^*$ and $R_{11}[\xi_\tau,\Delta z^*]$ obtained by keeping probe B location fixed at ξ^* and using the time-delay and the convection velocity information (Taylor's hypothesis). The agreement is very good, in figure 2, for $\Delta z^* \geq 0.20$; and $\Delta z^* > 0.40$ the two-methods do not produce close-enough correlation, which, however, is expected as in that case, due to a large separation, the big eddies sensed by the upper probe are likely to be distorted before being sensed by the lower probe and thus the assumptions of the frozen turbulence break down.

There is another way to look at the phenomenon of applicability of Taylor's hypothesis: if the profiles of $R_{11}[\xi,\Delta z^*]$ corresponding to various Δx^* for a particular Δz^* are plotted on the same graph, then validity of Taylor's hypothesis implies that these profiles should collapse. This is shown in figure 3 for various Δz^*. The profiles plotted in figure 3 suggest that Taylor's hypothesis is more applicable for smaller Δz^* as the profiles corresponding to $\Delta z^* = 0.07$ for various Δx^* collapse almost on a single curve.

With reference to figure 3, it can be reasonably said that Taylor's hypothesis can be used in two-point correlation measurements up to a wall-normal separation Δz^* of the order of 0.27. Beyond this limit the profiles corresponding to different Δx^* do not collapse to a single curve. However, this may not restrict the use of Taylor's hypothesis in deducing the inferred structure angle for larger probe separations as ξ_m is somewhat invariant with the streamwise separations. The lack of collapse of various Δx^* profiles at larger Δz^* may be indicative of any distortion that a big eddy is undergoing as it is convected downstream. With regard to the applicability of Taylor's hypothesis in cross-correlation measurements, it is appropriate here to mention the work of Favre et al. [3] where they compared the auto-correlation (deduced using Taylor's hypothesis with $U_c = U$) and the longitudinal (streamwise) space-correlation (obtained using two probes separated in the streamwise direction only). Favre et al. [3] presented measurements at four different z/δ_H positions of 0.03, 0.15, 0.24 and 0.77 in the boundary layer. Their results show that the difference between the auto-correlation and the longitudinal space-correlation is negligible (when one is transformed to the other using Taylor's hypothesis) at $z/\delta_H = 0.24$. But for $z/\delta_H = 0.77$ Taylor's hypothesis overestimates the correlation (when using the auto-correlation to obtain the space-correlation), while for $z/\delta_H = 0.15$ and $z/\delta_H = 0.03$ it underestimates the correlation. This is possibly because of a wrong choice of the average convection velocity. From these observations one might intuitively suggest a convection velocity which is equal tot he local mean velocity at $z/\delta_H = 0.24$ but larger than the local mean velocity for $z/\delta_H < 0.24$ and smaller than the local mean velocity for $z/\delta_H > 0.24$. This argument is also consistent with the argument of Sternberg [7] that the large eddies travel downstream at a speed characteristics of the boundary layer as a whole rather than at a local mean velocity. That is, in other words, in the outer portion of the boundary layer, these eddies should be convected more slowly than the mean velocity, and in the inner portions more rapidly than the local mean velocity. This makes $U_c = 0.82U_1$ an acceptable choice for the convection velocity.

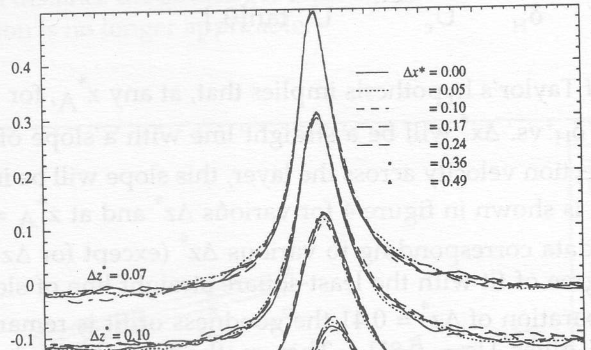

Fig. 3: Validity of Taylor's hypothesis for two point correlation measurements. The profiles represent cases where ξ is a mixture of time-shift t and space-shift Δx, and correspond to various Δz^*. Note the shift in ordinates

There is yet another alternative way to look at the question of the validity of Taylor's hypothesis in the case of interpreting average structure angle from correlation measurements. Rewriting (4), one obtains

$$\tan[\theta_i] = \frac{\Delta z^*}{\xi_m} = \frac{\Delta z^*}{\Delta x^* - \dfrac{\tau_m U_c}{\delta_H}}$$

Rearranging the terms

$$\frac{\tau_m U_1}{\delta_H} = \frac{U_1}{U_c}\Delta x^* - \frac{U_1}{U_c}\frac{\Delta z^*}{\tan[\theta_i]} \qquad (5)$$

The validity of Taylor's hypothesis implies that, at any z^*_A, for a particular Δz^*, a plot of $\tau_m U_1/\delta_H$ vs. Δx^* will be a straight line with a slope of U_1/U_c and for a constant convection velocity across the layer, this slope will be invariant with Δz^*. One such plot is shown in figure 4 for various Δz^* and at $z^*_A = 0.26$. In figure 4, experimental data corresponding to various Δz^* (except for $\Delta z^* = 0.31$) shows a very good degree of fit with the least-square straight line of slope 1.25. Even for the largest separation of $\Delta z^* = 0.41$ the goodness of fit is remarkable. A slope of $U_1/U_c = 1.25$ gives $Uc = 0.8U_1$. This method of determining the average convection velocity has one distinct advantage over the popularly employed technique of predicting convection velocity using two probes with streamwise separation only (e.g. see Favre et al. [3], Wills [11]. Irrespective of careful design of the upstream probe, the organized structures are very much likely to be disturbed by the probe arrangement as used by the latter scheme. From the measurements presented in figure 4, it can be conjectured that Taylor's hypothesis can be used with reasonable confidence for the measurements of the inferred structure angle using two-point space-time cross-correlations. Also a convection velocity of $0.82U_1$ which is constant throughout the boundary layer is a reasonable choice. In this regard the estimation of U_c from space-time correlation measurements of pressure fluctuations (defined as R_{pp}) at a wall under a thick turbulent boundary layer by Willmarth andWooldridge [10] can be mentioned. By filtering the measured R_{pp} in several frequency bands they deduced that $U_c = 0.83U_1$ for pressure-deducing large eddies and $U_c = 0.69U_1$ for pressure-producing small eddies. These results are in a good agreement with the findings of the present study by a different approach. Note that the probe separation in the present study acts as a spatial filter and it appears that only the large eddies are being correlated. However, it is conceded here that, because of lack of profiles closer to the wall, the correspondence between U_1 and U_c cannot be confidently conjectured for z/δ_H

very close to the wall. It can be mentioned here parenthetically that some investigations based on the direct numerical simulation (DNS) data suggest that Taylor's hypothesis breaks down in the region very close to the wall.

Figure 3 presents a few other interesting features. It appears that the structures show positive correlation for a distance approximately equal to $2.5\delta_H$ implying a correlation width a approximately $5\delta_H$. Although ξm moves towards higher values as Δz^* is increased, the distance between the peak correlation and the end of the positive correlation always remains of the order of $3\delta_H$. Note that for any separation, when the correlation goes close to zero or drops below zero, the profiles corresponding to various Δx^* do not collapse all that well. This indicates that after a certain distance the eddies get disintegrated and as such the hypothesis of frozen convection is no longer applicable.

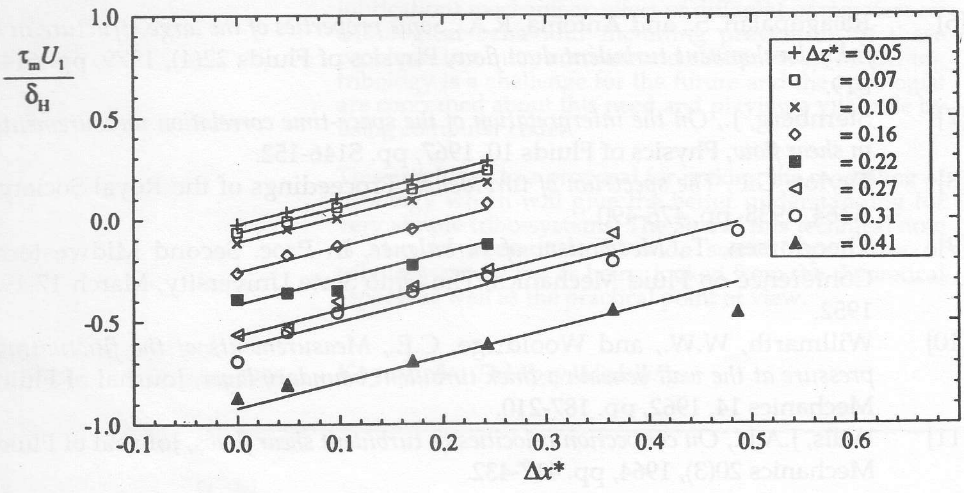

Fig. 4: An alternate way to examine the validity of Taylor's hypothesis. Solid lines represent straight line with a slope of 1.25

CONCLUSION

It appears from the experimental study present here that the use of Taylor hypothesis in a turbulent boundary layer, with a constant convection velocity along the whole layer, is acceptable. A value of $U_C = 0.82U_1$ is a reasonable estimate for the convection velocity. As such, Taylor hypothesis can be used, in conjunction with two-point space-time correlation measurements, in the identification of organized structures in a boundary layer.

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