A Numerical Study on the Water Entry of Two Dimensional Ship-like Sections

M. Reaz H. Khondoker
Department of Naval Architecture and Marine Engineering, Bangladesh University of Engineering and Technology
Dhaka, Bangladesh.

Abstract: In veiw of the importance of impact force estimation during the slamming of ships in waves and the water entry of free-fall lifeboats, this paper is devoted to the basic study of water entry of two dimensional ship-like sections. For the purpose, the basic theory of water impact has been reviewed and the added mass of wedged shaped section proposed by different authors are explored. Numerical study has been carried out to find the differences in behaviour during water entry of wedge section due to the use of various formulations of added mass. Moreover, the mechanism of water entry is investigated for different types of sectional model based on the momentum theory, the added mass concept and assumption on the irreversible nature of the impact. It is seen from the study that the generated acceleration during the water entry of concave section is higher than those of convex and wedge shaped sections if the areas of the sections are not much different.

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INTRODUCTION

The general problem of hydrodynamic impact of rigid or elastic structures has been extensively investigated in several fields such as landing of seaplanes and space craft capsule on the sea surface, launching of torpedoes and missiles, slamming of ships and offshore structures, liquid sloshing inside tanks and impact of breaking waves on coastal structures. In the field of naval architecture and ocean engineering, attention is given to the phenomena of water entry of a free-fall lifeboat very recently. Since many marine accidents are reported during the lowering of conventional lifeboat from a listing vessel, the importance of free-fall lifeboat as a lifesaving equipment increases rapidly. In many sea going vessels and offshore drilling platforms also these are being used due to the simplicity of launching method and quick moving away from the danger zone.

Analysis of the water impact loads during slamming of ships and water entry of free-fall lifeboats is an extremely difficult task because in rough seas there are several factors which can affect the hull structure subjected to water impact. As the impact load is affected by sectional shape of the ship hull, it is important to study the relationship between hull geometry and impact load. Due to the importance of impact force estimation of the sections of ships and lifeboats, this paper is devoted to the basic study of water entry of two dimensional ship-like sections. In the following, the basic theory of water impact has been reviewed and the added mass of wedged shaped section proposed by different authors are explored. Numerical simulation has been carried out to find the differences in behaviour during the water entry of wedge with various added mass formulations. Moreover, the mechanism of water entry is investigated for different types of sectional model based on the momentum theory, the added mass concept and the assumption on the irreversible nature of the impact.

MOMENTUM THEORY

von Karman proposed his theory for the vertical water entry of a cylinder based on the conservation of momentum. The section of a cylinder as shown in Fig. 1 is considered. When the section penetrates the water surface water pressure will act on the section, which when integrated, can be interpreted as inertia forces, damping forces and buoyancy forces. In this case the inertia force can be found by differentiating the momentum transfer.

Assuming that the mass per unit length of the cylinder or the sectional mass is M_O and the vertical penetration of the section into the water initially at rest to be z, the velocity of the section is $v(t) = z$. If the initial downward velocity is V_O, then the initial downward momentum of the section is $M_O V_O$. After some time t, at the corresponding immersion z(t), the velocity of the body will be reduced to v(t). The original momentum of the section $M_O V_O$ will be shared between the section and the surrounding water. Therefore, conservation of momentum requires:

$$[M_O + m_a(z)]\, v(t) = M_O V_O \qquad (1)$$

where $m_a(t)$ is the added mass of the section. Since z(t) is a positive monotonous function, Equation (1) can be rewritten as:

$$[M_O + m_a(z)]\, v(z) = M_O V_O \qquad (2)$$

where $m_a(z)\ v(z)$ is momentum gained by the water. The added mass of the section $m_a(z)$ is a function of the immersion and this represents the hydrodynamic inertia felt by the section during the immersion.

The force on the section associated with the momentum transfer can be obtained by differentiating the momentum transfer:

$$F_m(z) = \frac{\partial}{\partial t}[M_o v(z)] = -\frac{\partial}{\partial t}[m_a(z)v(z)] \tag{3}$$

or by using Equation (1) to eliminate v(z)

$$F_m(z) = \frac{-M_o^2 . V_o \dfrac{\partial m_a}{\partial t}}{[M_o + m_a(z)]^2} \tag{4a}$$

$$F_m(z) = \frac{-M_o^3 . V_o^2 \dfrac{\partial m_a}{\partial z}}{[M_o + m_a(z)]^3} \tag{4b}$$

and the resulting deceleration of the section becomes:

$$a(z) = \frac{F_m(z)}{M_o} = \frac{-M_o^2 . V_o^2 \dfrac{\partial m_a}{\partial z}}{[M_o + m_a(z)]^3} \tag{5}$$

Hence by treating the impact at the water entry as an inelastic collision between the body and the added mass, its motion can be derived without cumbersome fluid dynamic calculations. If the added mass as function of the immersion is known, Equations (4) and (5) fully describe the impact forces and the motion of the body during the water entry. It is to be noted here that the effects of gravity, buoyancy and drag have been ignored in impact estimation. In the case of an entry with considerable velocity these forces are small compared to the force associated with the momentum transfer. However, as the entry progresses and the body loses most of its impact velocity, they gain importance and eventually dominate the motion.

It is clear that the added masses in the preceding equations are not constant. It is usually a function of the immersion of the body into the water, when immersion depends on the time. The dependence on the immersion is not linear and impact force changes due to this non-linearity.

ADDED MASS OF WEDGE SHAPED SECTIONS

Equations (4) and (5) show that the added mass m_a and its derivative are playing a significant role in calculating the impact force or force due to momentum transfer. Therefore, the proper estimation of the added mass is extremely important in the calculation of force due to momentum transfer or impact force.

Various approximations for the calculation of the added mass of wedge shaped sections have been proposed by different authors. von Karman [1] approximated the added mass of a section of wedge shaped cylinder (Fig. 1) by the added mass of a flat plate at the water surface; the width of the plate being the width of the wedge at the water surface. For a fully immersed long plate of width 2x, the theoretical added mass is prx^2, where r is the density of water. Thus, for a plate having water on one side only, a first approximation to the added mass is prx^2. This leads to the expression for the added mass for a two dimensional section as:

$$\text{for } 0<z<r \qquad m_a = \frac{1}{2}\pi\rho c^2 = \frac{1}{2}\pi\rho z^2 \cot^2\beta \tag{6}$$

Here, c is the half breadth of the wedge at the wetted surface, c_m is the maximum half breadth and β is the dead rise angle of the wedge (Fig. 1).

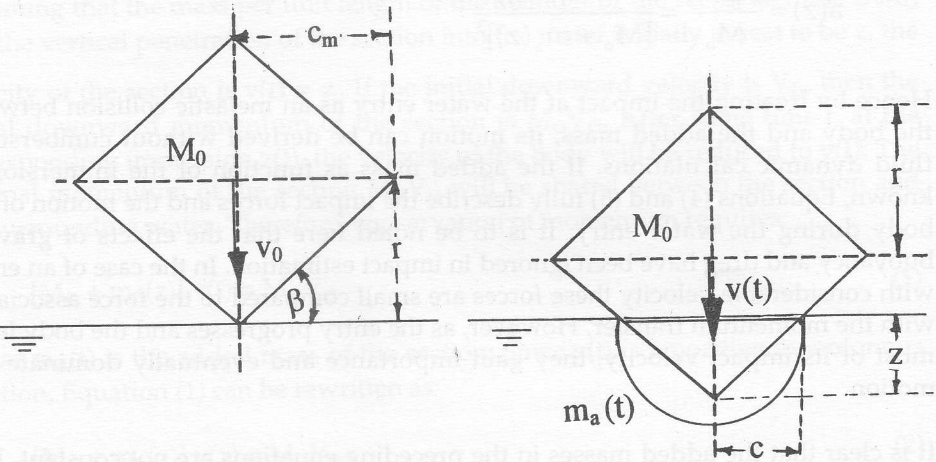

Fig. 1: Water entry of a cylinder with rhomboidal section

Wagner [2] refined this approximation taking into account the piling up of the water surface to determine the wetted breadth of the wedge. For small dead-rise angle he found that;

$$m_a = \frac{1}{8}\pi^3 \rho z^2 \cot^2 \beta \qquad (7)$$

Later on Wagner and Sydow [3] proposed a correction on the above expression which better approximates the added mass for large dead-rise angle,

$$m_a = \frac{1}{2}\pi \rho z^2 (\frac{\pi}{2\beta} - 1)^2 \qquad (8)$$

Kreps [4] proposed after a study of experimental data, the formula for added mass as;

$$m_a = \frac{1}{8}\pi^3 \rho \cot^2 \beta z^2 (1 - \frac{\beta}{\pi}) \qquad (9)$$

After studying data obtained from planning experiments, Mayo [5] inserted an empirically determined correction constant in Equation (8) as follows:

$$m_a = 0.41\pi \rho z^2 (\frac{\pi}{2\beta} - 1)^2 \qquad (10)$$

There have been few reliable experimental checks of the theories described above. In most cases, their verification has been based upon three dimensional experiments which were not well suited for checking fundamental two dimensional aspects of the problem. However Payne [6] presented a very interesting paper on the impact of wedge, which claims that the original approach by von Karman is superior to most of the so called "improved" formulations by other authors. Therefore to investigate the effect of added mass on the motions of different cross-sectional shapes, the added mass formulation of von Karman will be used here.

ADDED MASS OF RHOMBOIDAL SECTION

It is found from the added mass expressions in the preceeding section that these can be expressed by a general equation as follows:

$$m_a = \frac{1}{2} K z^2 \tag{11}$$

where $K = prcot^2b$ from Equation (6)

$K = \frac{1}{4}\pi^3 \rho \cot^2 \beta$ from Equation (7)

$K = \pi\rho(\frac{\pi}{2\beta} - 1)^2$ from Equation (8)

$K = \frac{1}{4}\pi^3 \rho \cot^2 \beta (1 - \frac{\beta}{\pi})$ from Equation (9)

$K = 0.82\pi\rho(\frac{\pi}{2\beta} - 1)^2$ from Equation (10)

Now, for one sided immersion of a rhomboidal section, as shown in Fig. 1, Equation (11) can be used to estimate the added mass and its derivative can be derived from the same relation by differentiating the added mass. The expressions for added mass, m_a, its derivative, dm_a/dz and immersed area, $A_i(z)$ can be given as:

$$m_a = \frac{1}{2} K z^2 \tag{12a}$$

$$\frac{dm_a}{dz} = Kz \tag{12b}$$

$$A_i(z) = z^2 \, cotb \tag{12c}$$

For deeper immersion of the wedge ($z/r>1$) the flat plate hypothesis can no longer be used. By assuming that the added mass at $z/r = 2$ is equal to the added of a fully submerged plate and using a linear interpolation between $z/r = 1$ and $z/r = 2$, Boef [7] obtained the following expression:

$$m_a = \frac{1}{2} K r z \tag{13a}$$

$$\frac{dm_a}{dz} = Kr \tag{13b}$$

$$A_i(z) = 2r^2 \cot\beta [1 - \frac{1}{2}(2 - \frac{z}{r})^2] \tag{13c}$$

ADDED MASS OF SHIP-LIKE SECTIONS

The geometry of transverse sections of ships with convex, wedge and concave shape as shown in Fig. 2, can be expressed by a general equation (for $z<r_1$) as:

$$c = kz^n \tag{14a}$$

$$\frac{dc}{dz} = nkz^{n-1} \tag{14b}$$

where, z is the immersion at a certain time, c is the half-breadth at immersion z and k is a coefficient depends on maximum half-breadth (c_m), maximum depth (r_1) of the lower portion and a coefficient n, when

if n<1, the section is of convex shaped
if n=1, the section is of wedge shaped
if n>1, the section is of concave shaped

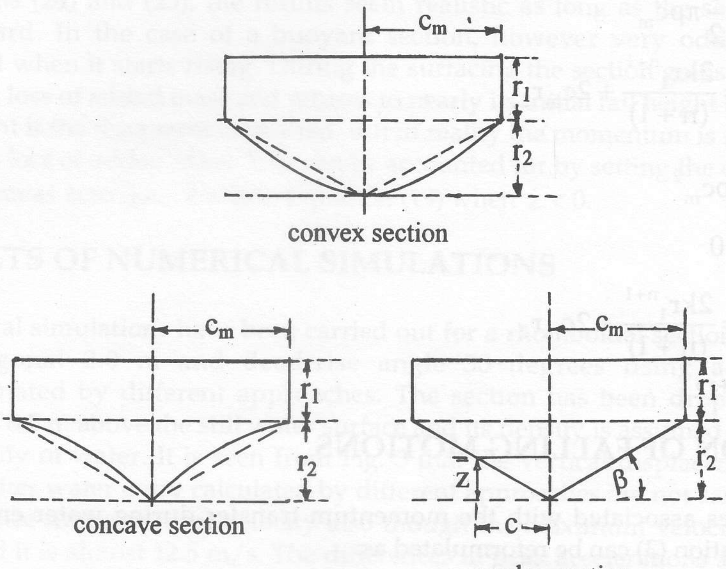

Fig. 2: Ship sections with convex, wedge and concave shape

Now, the added mass and its derivative for these types of sections can be written as: for $0<z<r_1$:

$$m_a = \frac{1}{2}\pi\rho c^2 \tag{15a}$$

$$\frac{dm_a}{dz} = \pi\rho c \frac{dc}{dz} \tag{15b}$$

$$A_i(z) = \frac{2kz^{n+1}}{(n+1)} \tag{15c}$$

for $r_1<z<R$:

$$m_a = \frac{1}{2}\pi\rho {c_m}^2 \tag{16a}$$

$$\frac{dm_a}{dz} = 0 \tag{16b}$$

$$A_i(z) = \frac{2k{r_1}^{n+1}}{(n+1)} + 2c_m(z-r) \tag{16c}$$

for $R<z<2R$

$$m_a = \frac{1}{2}\pi\rho z c_m \tag{17a}$$

$$\frac{dm_a}{dz} = \frac{1}{2}\pi\rho c_m \tag{17b}$$

$$A_i(z) = \frac{2k{r_1}^{n+1}}{(n+1)} + 2c_m r_2 \tag{17c}$$

or for $z>2R$

$$m_a = \pi\rho {c_m}^2 \tag{18a}$$

$$\frac{dm_a}{dz} = 0 \tag{18b}$$

$$A_i(z) = \frac{2k{r_1}^{n+1}}{(n+1)} + 2c_m r_2 \tag{18c}$$

where, $R=r_1+r_2$

EQUATION OF FALLING MOTIONS

For the forces associated with the momentum transfer during water entry of a section, Equation (3) can be reformulated as:

$$F_m(z) = -\frac{\partial}{\partial t}[m_a(z)v(z)] = -\frac{\partial}{\partial t}[m_a\dot{z}] \tag{19}$$

$$F_m(z) = -\dot{m}_a\dot{z} - m_a\ddot{z} = -\frac{dm_a}{dz}(\dot{z}^2) - m_a\ddot{z} \tag{20}$$

The other forces acting on the section are the gravity, buoyancy and drag. Their respective contribution can be expressed as:

$$F_g = M_o g \tag{21}$$

$$Fb = -rgA_i(z) \tag{22}$$

$$F_d = -c_d(z)\pi\rho.\dot{z}|\dot{z}| \tag{23}$$

In Equation (23) $c_d(z)$ is the drag coefficient which depends on immersion z. Hence the equations of motion for a section dropped on a water surface at $z = 0$ can be described by:

$$M_o.\ddot{z} = M_o.g \qquad \text{for } z<0 \tag{24}$$

$$M_o.\ddot{z} = M_o.g + F_b + F_d + F_m \qquad \text{for } z\geq 0 \tag{25}$$

If the motion of the section dropped from a height H is calculated by solving Equations (24) and (25), the results seem realistic as long as the section moves downward. In the case of a buoyant section, however very odd results are obtained when it starts rising. During the surfacing the section gains momentum from the loss of added mass and returns to nearly its initial fall height or exactly its fall height if the drag term is ignored. But in reality the momentum is not regained from the loss of added mass. This can be accounted for by setting the derivative of immersion as zero (i.e., $\dot{z} = 0$) in Equation (19) when $\dot{z} < 0$.

RESULTS OF NUMERICAL SIMULATIONS

Numerical simulations have been carried out for a rhomboidal section having the half-diagonal 2.0 m and dead-rise angle 30 degrees using added mass approximated by different approaches. The section has been dropped from a height of 8.0 m above the still water surface and its density is assumed to be half of the density of water. It is seen from Fig. 3 that the vertical displacements of the section after water entry calculated by different approaches are not same. There is a difference in the vertical velocity also though the maximum velocity is almost same and it is almost 12.5 m/s. The differences in peak accelerations are also very clear from the figure. It is to be noted that the acceleration time history has been shown only for 1.0 to 2.0 seconds since impact occurs within this period and after that the acceleration is not changing so much. It is seen that the peak acceleration obtained from Wagner's approximations is the maximum and that from von Karman's one is the minimum.

Figure 4 shows displacements, velocities and accelerations for three ship-like sections having convex, wedge and concave shape but each one is symmetrical around the vertical axis. The maximum half breadth (c_m) and the height of the rectangular part (r_2) have been used as same as the depth of the lower part (r1) and the value is 2.0 m. The value of n for convex section has been assumed to be 0.75, for wedge section 1.0 and for concave section 1.25. Therefore, there are not much differences in the area and mass of these sections. These are also dropped from the height of 8.0 m above the still water surface. The differences in the time history of displacements and velocities are not very clear though the differences in accelerations are clear. But the acceleration of the concave section is the maximum and that of the convex section is the minimum since the force due to momentum transfer and as well as the total force is the maximum for concave section and the minimum for the convex section respectively.

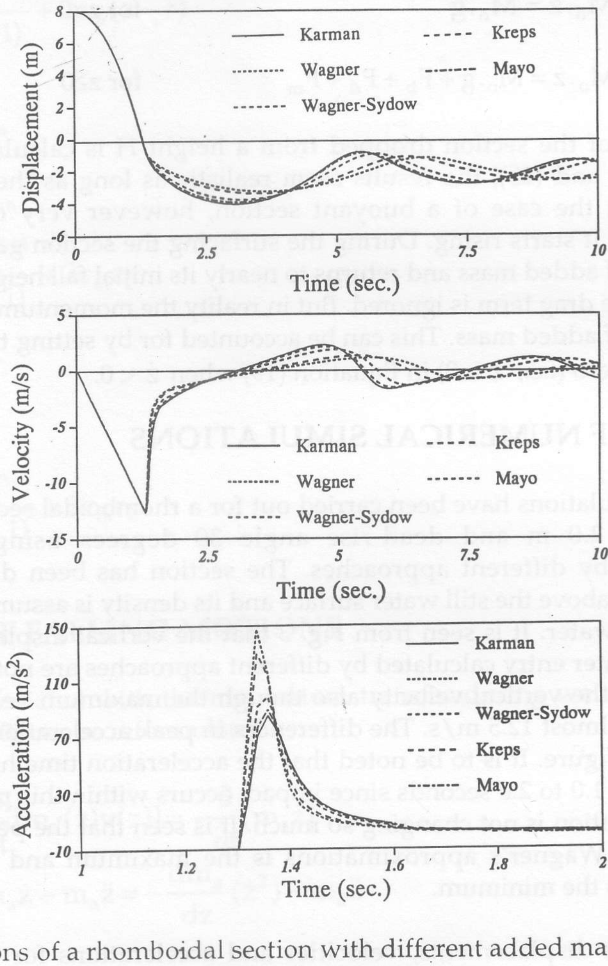

Fig. 3: Motions of a rhomboidal section with different added mass formulations ($r = 2.0$ m, $\beta = 30$ degree)

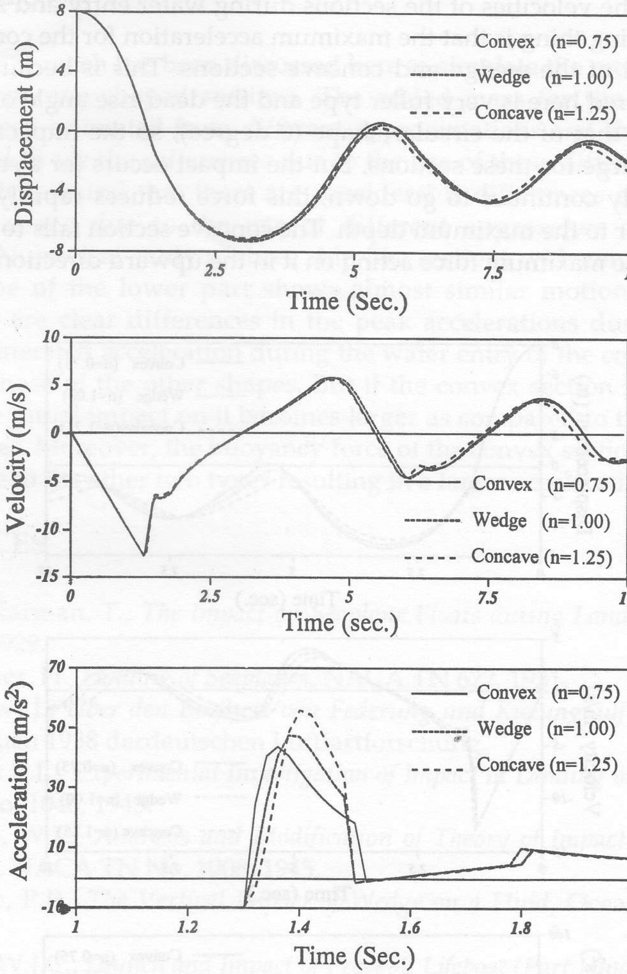

Fig. 4: Motions of ship sections with different hull shapes ($r_1 = 2.0$ m, $r_2 = 2.0$ m and $c_m = 2.0$ m)

Figure 5 shows displacements, velocities and accelerations for three different sectional shapes of ships, i.e., convex, wedge and concave. The depth of lower part has been taken as 2.0m, the height of the rectangular part is 1.5m and the maximum half breadth is 3.46m. The value of n for convex section has been considered to be 0.5, for wedge section, 1.0 and for concave section, 2.0. Therefore, there are considerable differences in the areas and masses of these sections. These are also dropped from a height of 8.0m above the still water surface. The differences in the time history of displacements are very clear and there are some differences in the velocities of the sections during water entry and surfacing also. But the interesting thing is that the maximum acceleration for the convex section is more than that of the wedge and concave sections. This is because the convex section considered here is very fuller type and the dead-rise angle of its tangent is almost near to that of the circular shape (0 degree), so the impact just at water touch is very large for these sections. But the impact occurs for a small time only and as the body continues to go down, this force reduces rapidly enabling the sections to enter to the maximum depth. The concave section falls to the minimum depth due to the maximum force acting on it in the upward direction.

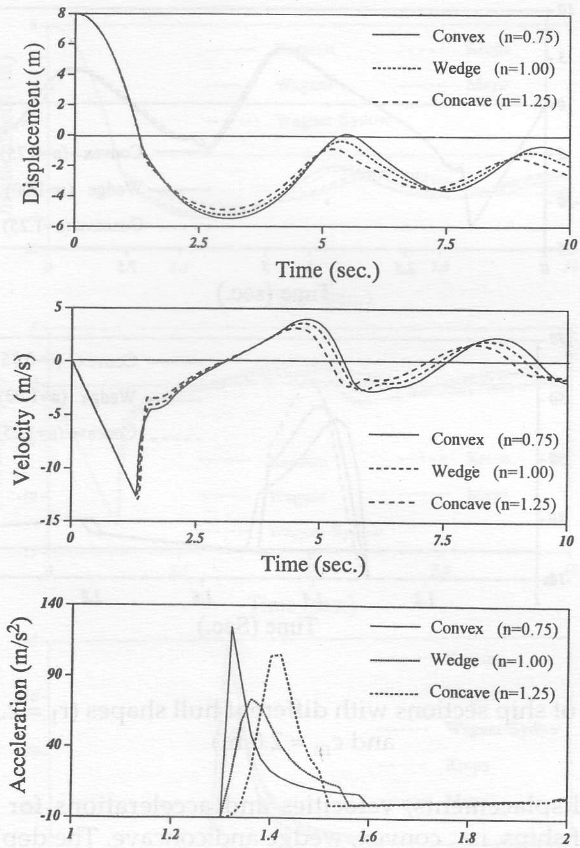

Fig. 5: Motions of ship sections with different hull shapes ($r_1 = 2.0$ m, $r_2 = 1.5$ m and $c_m = 3.46$ m)

CONCLUSIONS

A numerical approach has been discussed here to simulate the motions of convex, wedge and concave shaped sections. The added mass for the wedge shaped section has been estimated from different approximations and the motions have been analyzed to see the differences due to the use of these different added mass expressions. It is found that there are considerable differences especially in the peak accelerations due to the use of different approaches of added mass calculations. Numerical study of three ship-like sections with convex, wedge and concave shape of the lower part shows almost similar motion characteristics though there are clear differences in the peak accelerations due to the impact forces. The generated acceleration during the water entry of the concave section is higher than those of the other shapes. But if the convex section is of very fuller type, then the initial impact on it becomes larger as compared to that of wedge or concave shapes. Moreover, the buoyancy force of the convex section becomes very large compare to the other two types resulting in a larger acceleration.

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