Mechanical Behaviour of Jute-Glass Fibre Reinforced Composite Laminates

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Abstract : The experimental tensile strength and stiffness properties of Jute-Glass Reinforced Composite Laminates (JGRCL) at different volume fractions of jute and glass fibres are presented. It is observed that the strength and stiffness of JGRCL vary linearly with the "effective" fibre volume fraction. The experimental data are then correlated with Bishop's model using a least square method and the empirical parameters necessary for predicting the strength and stiffness of JGRCL at an arbitrary effective fibre volume fraction are evaluated. The experimental values of the strength and stiffness of JGRCL are compared with those predicted by lamination theory and law of mixtures. The flexural moduli of JGRCL obtained form experiment at different effective fibre volume fractions are then presented and compared with those predicted by lamination theory. It is observed that the predicted values of flexural moduli do not agree well with the experimental values.

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INTRODUCTION

Fibre reinforced resin composites have experienced growing number of engineering applications (e.g. aerospace, pressure vessels, boa hulls, bearings, etc.) over the last several decades because they possess superior physical properties such as high strength to weight ratio, insulating properties, etc. which metals lack. Laminated fibrous composites are most common among these applications. Design of such laminates for a particular appliction requires knowledge of strength and stiffness properties of the desired composite material at different fibre volume fractions, orientations, and lamina stacking sequence. However, experimental determination of these properties is possble only for a few combinations of the design variables (.e. fibre volume fraction, fibre orientation, and lamina stacking sequence). For optimal design, it is necessary to know or be able to predict the required mechanical properties for any combination of the design variables. An extensive body of literature exist on the analysis and prediction of the mechanical properties of fibre reinforced laminates. Netting analysis [6], which ignores strength contributions of the matrix, has been introduced for predicting the mechanical properties of fibrous felts and papers. However, for unidirectional composites, this method predicts zero strength in the direction perpendicular to the fibre which is untrue. Modifications [2,9,17] to the netting analysis have been proposed to account for the fibre-matrix interaction. Law of mixtures and the effects of matrix have been incorporated in these models. However, these analyses are inapplicable to brittle matrices because of the assumption that the matrix is ideally elastic-plastic. Based on the netting analysis, Bishop [4] has proposed an improved method for predicting the mechanical properties of fibre composite materials. This model incorporates the indirect fibre contributions in transverse directions by introducting two hypothetical "Lateral Fibres". The predicted stress-strain behaviour of a glass-fibre/epoxide-resin laminate and a glass-fibre pressure vessel under simultaneous bending and internal pressure are shown to be in good agreement with the observed data.

Law of Mixtures type model has been suggested [12] for predicting the strength of fibre composite materials. A uniform state of strain is assumed to exist as the load is applied until fracture initiates. It has been observed that failure of the composite occurs as soon as this uniform strain reaches the failure strain of the fibres. However, this model predicts too large a composite tensile strength as compared to experimental data.

Whitney and Riley [20] have proposed an analysis of a unidirectionally fibre reinforced composite based on the theory of elasticity. The theoretical predictions of the transverse and longitudinal moduli are shown to be in good agreement with the experimental values. However, the predicted shear modulus is significantly different from experimental vlaues. A finite-difference model [18], also based on the theory of elasticity, has been suggested. This model allows the longitudianl modulus and major Poisson's ratio to be predicted from the law of mixture formula and concentrates on better prediction for the transverse and shear modulus. The finite difference model, although yields better predictions, is relatively complex.

Use of variational techniques for predicting composite material properties has been reported by some authors [11]. It has been established that the longitudinal stiffness and the major Poisson's ratio are linear functions of fibre volume fractions supporting the validity of the Law of Mixtures. The classical lamination theory for multilayer laminates [7,14,15] embodies a collection of stress and deformation hypothesis and has been suggested for prediction purposes. Laminate stiffness predicted by this theory are observed to be in good agreeement with the experimental data [3,19].

Boue [5] has established that the fibre to matrix volume ratio has an effect on the failure mode of fibre-reinforced composites. He has observed that the failure of the specimens with low fibre volume fraction commences by transverse resin cracking. This is followed by fibre fracture and fibre pullout from both sides of the resin crack. For the high fibre volume fraction specimens, random fibre failure occurs when the applied load is below 50 percent of the ultimate load. The failure of the composite occurs by an accumulation of random fractures.

Zweben [21] has propsed a noncumulative fracture model for the tensile strength of fibre composites. Accoridng to this model, failure of composite occurs after the first fibre fails or at most a few isolated fibres fail. This model finds a correlation between theoretical strength of the weakest fibre and the observed composite failure loads. This model has shown reasonable agreeemnt with experimental tensile strength for a class of fibre composites. Cumulative Weakening Model [10] has been propsed to account for the failure modes of brittle composites. In this model, the composite is divided into a series of layers of unit thickness. Thus the composite becomes a chain of bundles. However, this model does not agree well with the expeirmental results. Fariborg, et al [8] have investigated the tensile behaviour of intraply hybrid composites. They modified the basic chain of bundles probability model. The existence of the hybrid effect on strian is established and its sensitivity to volume ratio and dispersion of the type of fibres is also discussed.

Kazim [13] has reported experimental determination of a number of mechanical properties of jute fibre reinforced composite laminates at different fibre volume fractions. A netting analysis type model has been adopted for the prediction of laminate properties. However, this analysis is less effective for cross-ply jute reinforced plastics.

The widespread application of Jute-Glass fibre reinforced palstics in structural components as well as consumer products in Bangladesh has necessitated a systematic study of the mechanical properties of this composite material and their predictions for different combinations of fibre volume fratgion, fibre orientation and lamina stacking sequence. Anwar [1] has investigated the experimental deterimination and subsequent prediction of the strength and stiffness properties of jute-glass fibre reinforced composite laminates (JGRCL). It is important to note that such studies are essential for the design and analysis of components made of hybrid fibre composite materials such as JGRCL.

Present paper starts with the experimental determination of the strength and stiffness properties of Jute and Glass fibre reinformced composite laminates. The experimental observations are then compared with the predicted values of strength and stiffness properties using three different models (e.g. Bishop's model, Lamination theory, and Law of Mixtures) for various combinations of effective fibre volume fraction. The effect of increasing the number of jute fibre lamina on the composite strength is discussed in section 3. Finally the experimental and predicted values of flexural moduli of JGRCL are presented and compared.

EXPERIMENTAL PROCEDURE

ASTM standard specimens for tensile and flexure tests are prepared using hand lay up method. Tensile tests of the JGRCL specimens are carried out at constant strain rate. A brief description of specimen preparation and testing procedure follows.

Preparation of Specimens

Woven cross-ply BTA grade jute fibre mats and E-glass fibre mats are cut into sizes of 254 x 508 mm. Laminates are made using hand lay-up method. Care is taken to make sure that no air gap remains within the laminate. For multilayer laminates, extra fibre mats are added to the first one while pouring polyester resin on each addition of mats. Jute fibre mats and E-glass fibre mats are placed as symmetrically as possible across the laminate cross-section to ensure even distribution of the strength properties. Non-reacting polythene sheets are placed on both sides of the laminate and pressure is exerted using smooth flat plates to further drive any remaining trapped air out.

A number of laminates are made using various combinations of jute and glass fibre mats. The fibres of all the layers of a laminate are aligned in either 0^o or 90^o directions along the length of the specimens. The laminates are then cured slowly at room temperature and under little pressure. After solidification of the resin, each laminate is weighed. The weight of the resin is obtained by subtracting the weight of fibre mats from total weight of the laminate. Using the density values of the constituent materials, the fibre volume fractions of jute and glass fibre are claculated.

Ten tensile test specimens, each of dimension 254 mm in length and 25.4 mm in width, are made from each type of laminate according to the standards specified by ASTM D 3039-76. Each specimen is then polished using a fine grinding wheel in order to avoid any stress concentration effect at the edges. Tabs of size 38.1 x 25.4 mm are also made accoridng to the ASTM specifications which are then attached to the ends on both sides of each specimen using a very strong adhesive (Araldite).

Flexure test specimens are prepared in the same way as described above. Here wovern cross-ply glass and jute fibre mats are sized 15 cm in length and 4 cm in width. Multilayered laminates are made for various combination jute and glass fibre layers with thicknesses raging from 1.3 mm to 2.2 mm. The laminates are cured uder mild pressure for two days in order for the laminate to develop full strength. Ten specimens each of dimension 40 mm in length and 14 mm in width are made from each type of laminate having a particular combination of jute and glass fibre volume fractions. These specimens are then polished using a fine grindng wheel.

Testing Method

Different dimensions of each tensile test specimen (e.g gage length, width, minimum thickness, etc) are measured before begining the test. Tensile test of each specimen is then conducted using a "Universal Tensile Testing Machine" at Bangladesh Educational Equipment Board (BEEB) laboratory at room temperature and without any initial stress. The straining rate has been approximately kept constant (between 0.01 mm/mm.min and 0.02 mm/mm.min) during each test. Load is applied in the direction curve is automatically recorded on a chart. The elongation at failure is obtained from the chart. Young's Modulus for each specimen is then calculated using the load-elongation curve. Ten specimens from each type of laminate have been tested.

The flexure test specimens are tested in a "Flexural Modulus Measuring Apparatus" at BITAC (Bangladesh Industrial and Technical Assistance Centre). At the begining of the experiment, the mean thickness d of each specimen over its full width at the mid section is measured and recorded. The maximum strain of each specimen should be kept below 0.2 percent in order to ensure elastic deflection.This value of critical strain is based on the stress-strain properties of the specimens. The maximum allowable deflection D at the mid-section of each specimen is calculated from the following equation:

$$D = \frac{0.21505}{d} \qquad (1)$$

The specimens are tested on the apparatus using three-point test method. Each specimen is placed centrally on the suports and then the load beam is placed on the middle of the specimen. Loose weights are applied at the centre of the beam successively until the dial gauge reading becomes approximately 2D. The applied load W is recorded. Exactly one minute after the completion of loading, the resultant deflection D is measured to the nearest 0.002 mm and recorded. The remaining specimens are tested using the above procedure.

The flexural modulus E of each specimen is calculated from the following euation (based on the beam deflection theory) :

$$E = \frac{L^3W}{4d^3Db} \qquad (2)$$

Where b = Specimen width
L = Specimen span length
W = Applied load

RESULTS AND DISCUSSIONS

It has been observed by Boue [5] that the mechanical properties of a fibre composite are functions of the fibre volume fraction of the composite. Thus knowledge of fibre volume fraction of a fibre composite is very important to predict its properties. Two types of fibres have been used for reinforcement of the composite laminate in the present investigation. Since the strength and stiffness properties of the composite depend on each of the fibre volume fractions, an "effective" fibre volume fraction can conveniently be defined in order to interpret these properties. Simple summation of the two fibre volume fractions is not desirable for obvious reasons. The effective fibre volume fraction of JGRCL to characterize a particular composition of the composite is defined as :

$$V_e^f = 2\frac{(V_f^j + V_f^g E_g)}{(E_j + E_g)} \qquad (3)$$

Basically the effective fibre volume fraction V_e^f is the weighted average of the two fibre volume fractions V_j^f (jute) and V_g^f (glass) based on their elastic moduli E_j and E_g which ensures a proportional increase in V_e^f with a small increase in a fibre volume fraction having higher modulus and vice versa. If the two types of fibre have the same elastic moduli, then the above expression for V_e^f becomes simple summation of the two individual fibre volume fractions.

Tensile test of Jute-Glass Reinforced Composite Laminate (JGRCL) has been performed for five different effective fibre volume fractions. The experimental and predicted Young's Moduli of the JGRCL for different volume fractions is plouced in Figure 1. It is observed that the experimental modulus increases linearly with effective fibre volume fraction of JGRCL. The maximum deviation between experimental stiffness values and those predicted by Bishop's model is about 5 percent which is expected because of the scatter in the experimental data. The maximum deviation of the stiffness values predicted by Lamination theory and Law of Mixture from experimental stiffness data is about 51 percent.

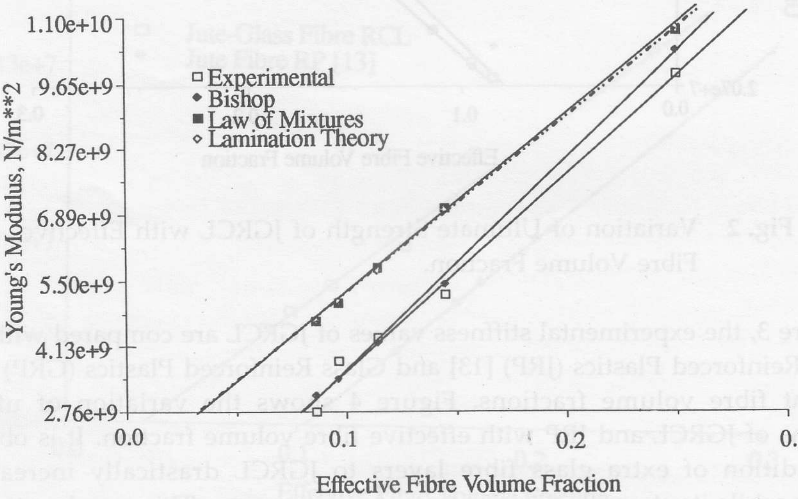

Fig. 1 Variation of Young's Modulus of JGRCL with Effective Fibre Volume Fraction.

The ultimate strength of JGRCL is observed to increase linearly with effective fibre volume fraction as shown in Figure 2. The maximum deviation between the ultimate strength predicted by Bishop's model and the corresponding experimental values is about 7 percent. A higher degree of scatter in the experimental data may have caused a little more deviation in the predicted strength in this case. The discrepancy of the predicted stiffness and strength values of JGRCL from experiment at higher volume fractions may be reduced using more experimental data. Voids and cracks have been assumed to be absent in the laminate while evaluating the empirical parameters of Bishop's model. This assumption may also have caused some discrepancies in the predicted properties, since voids are invariably present in the laminates made by hand lay-up method.

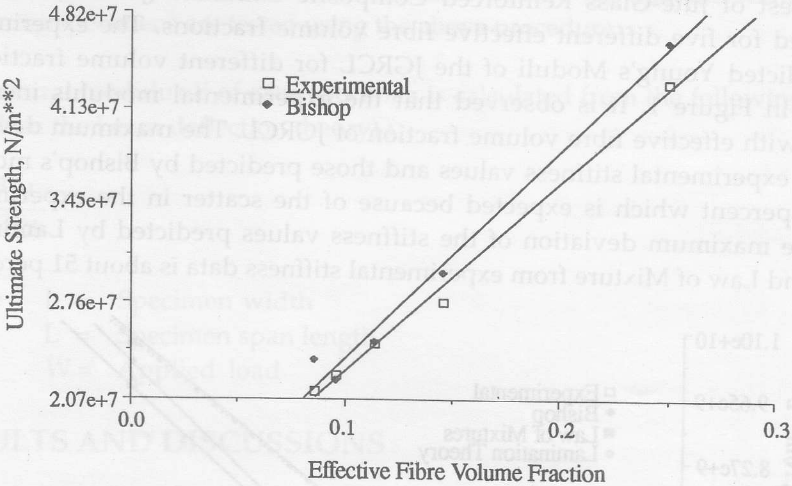

Fig. 2 Variation of Ultimate Strength of JGRCL with Effective Fibre Volume Fraction.

In Figure 3, the experimental stiffness values of JGRCL are compared with those of Jute Reinforced Plastics (JRP) [13] and Glass Reinforced Plastics (GRP) [16] at different fibre volume fractions. Figure 4 shows the variation of ultimate strengths of JGRCL and JRP with effective fibre volume fraction. It is observed that addition of extra glass fibre layers to JGRCL drastically increases its stiffness while its strength experiences a slower increase. This may be attributed to the fact that as more glass fibre layers are added to the laminate, the elongation of the specimen within elastic limit becomes very small resulting in a large increase in the elastic modulus. However, the strength of JGRCL has shown

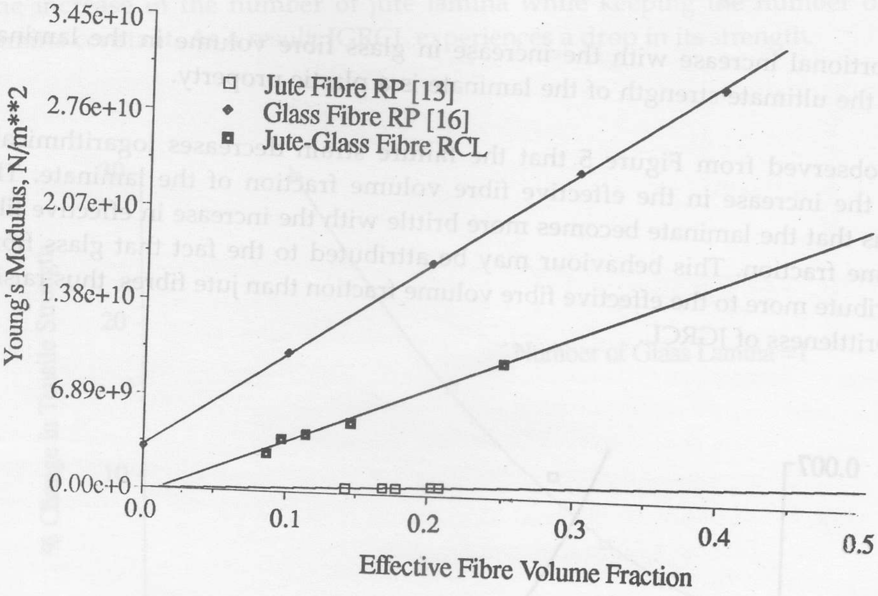

Fig. 3 Young's Moduli for JRP, GRP and JGRCL at different Effective Fibre Volume Fractions.

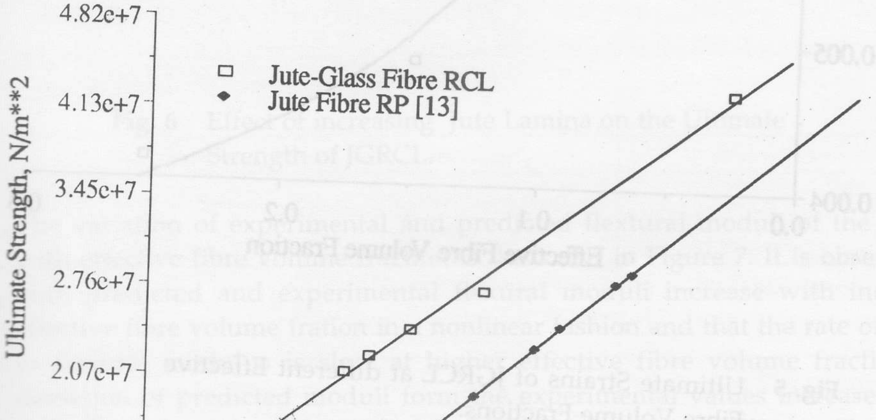

Fig. 4 Ultimate Strength of JGRCL and JRP at different Fibre Volume Fractions.

proportional increase with the increase in glass fibre volume in the laminate since the ultimate strength of the laminate is a plastic property.

It is observed from Figure 5 that the failure strain decreases logarithmically with the increase in the effective fibre volume fraction of the laminate. This means that the laminate becomes more brittle with the increase in effective fibre volume fraction. This behaviour may be attributed to the fact that glass fibres contribute more to the effective fibre volume fraction than jute fibres, thus raising the brittleness of JGRCL.

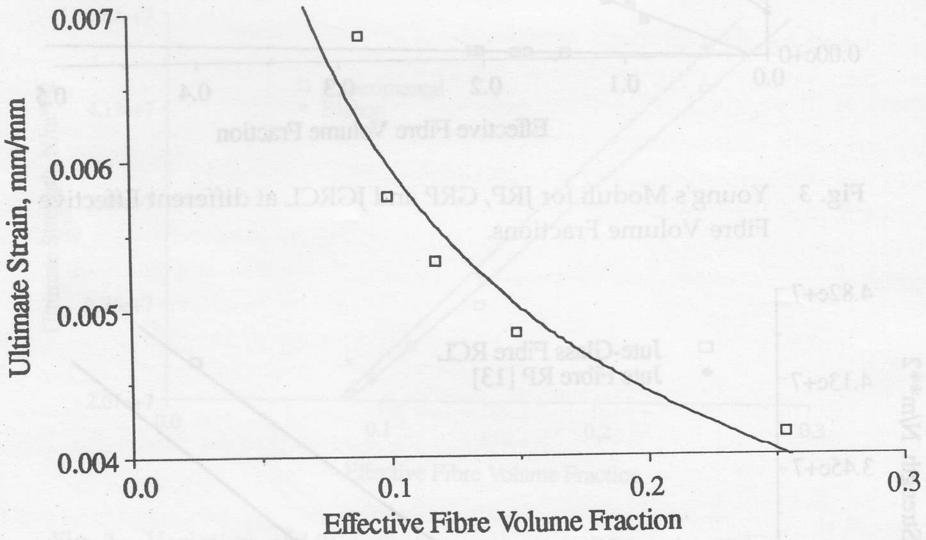

Fig. 5 Ultimate Strains of JGRCL at different Effective Fibre Volume Fractions.

Figure 6 shows the effect of increasing number of jute lamina on the strength of JGRCL while keeping the number of glass lamina constant. Jute fibre layers are placed symmetrically on each side of the glass fibre layer when the number of jute layers is even. For odd number of jute fibre layers, the cross-section symmetry in the laminate could not be maintained. It is observed that increasing the number of jute lamina decreases the strength of the laminate nonlinearly. This is expected because the effective voluem fraction of the laminate decreases with the increase in the number of jute lamina while keeping the number of glass lamina constant. As a result, JGRCL experiences a drop in its strength.

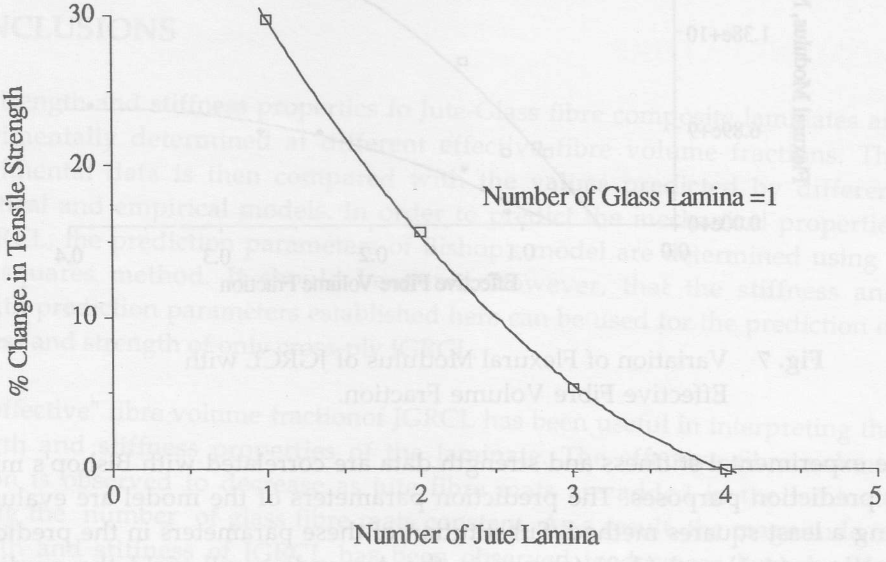

Fig. 6 Effect of increasing Jute Lamina on the Ultimate Strength of JGRCL.

The variation of experimental and predicted flextural moduli of the laminate with effective fibre volume fraction is illustrated in Figure 7. It is observed that both predicted and experimental flexural moduli increase with increase in effective fibre volume fration in a nonlinear fashion and that the rate of increase in flexural modulus is slow at higher effective fibre volume fractions. The deviation of predicted moduli form the experimental values increases rapidly with the increase in effective fibre volume fraction. At an effective fibre volume fraction of 0.386, the predicted modulus becomes about twice as much as the experimental value.

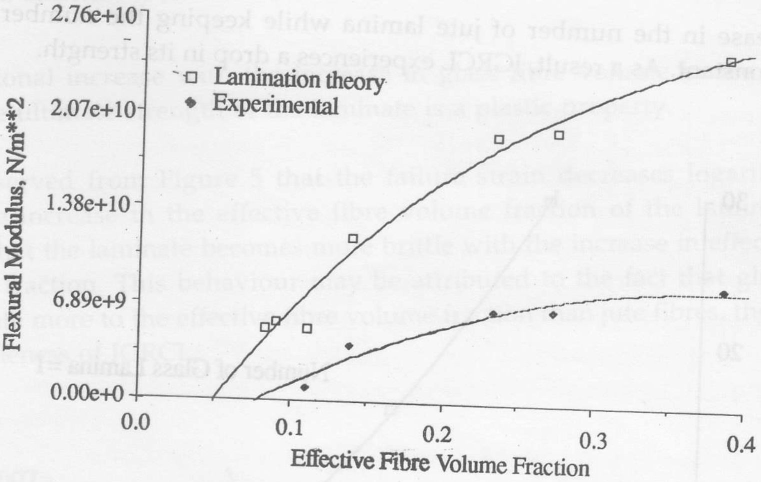

Fig. 7 Variation of Flexural Modulus of JGRCL with Effective Fibre Volume Fraction.

The experimental stiffness and strength data are correlated with Bishop's model for prediction purposes. The prediction parameters of the model are evaluated using a least squares method. Substitution of these parameters in the prediction equations (A.1) and (A.2) (shown in the Appendix) will yield the predicted values of longitudinal and transverse stiffnesses and strengths of cross-ply JGRCL. These stiffness and strength prediction parameters are tabulated below:

Prediction Parameters	Stiffness	Strength
Fibre Tensile, E_f	6.55×10^{10} N/m^2 (9.5×10^6 psi)	7.58×10^{10} N/m^2 (11×10^6 psi)
Lateral Fibre Stiffness E_1	7.58×10^7 N/m^2 (11,000 psi)	6.41×10^7 N/m^2 (9,300psi)
Matrix Tensile Stiffness, E_m	-6.68×10^6 N/m^2 (-970 psi)	-6.0×10^6 N/m^2 (-870 psi)
Empirical Constant, μ	899.223	898.23
Matrix Poisson's Ratio, ν	0.3	1×10^{-6}
Half Angle between Lateral Fibres, φ	35^o	35^o

The above parameter values can be substitituted in Eqs. (A.1) - (A.3) to obtain the predicted stiffness and strength values of cross-ply jute-glass fibre reinforced composite laminates at other effective fibre volume fractions.

CONCLUSIONS

The strength and stiffness properties fo Jute-Glass fibre composite laminates are experimentally determined at different effective fibre volume fractions. The experimental data is then compared with the values predicted by different analytical and empirical models. In order to predict the mechanical properties of JGRCL, the prediction parameters of Bishop's model are determined using a least squares method. It should be noted, however, that the stiffness and strength prediction parameters established here can be used for the prediction of stiffness and strength of only cross-ply JGRCL.

The "effective" fibre volume fractionof JGRCL has been useful in interpreting the strength and stiffness properties of the laminate. The effective fibre volume fraction is observed to decrease as jute fibre mats are added in the laminate keeping the number of glass fibre mats constant. As a result, the magnitude of strength and stiffness of JGRCL has been observed to decrease linearly. The brittleness of the laminate is observed to increase with the increase in effective fibre volume fraction.

Bishop's model has been identified to be the most effective model in predicting the strength and stiffness values of JGRCL. The maximum deviation between predicted and experimental values is found to be 7 percent for strength and 3 percent for stiffness. However, the stiffness values of the JGRCL predicted by the lamination theory as well as the law of mixtures are not in good agreeemnt with those obtained from experiment.

The flexural modulus of the laminate is observed to increase with the increase in effective fibre volume fraction. Although the lamination theory has predicted a large increase in the modulus at higher vibre volume fraction, the experimental values has not experienced that kind of increase.

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NOMENCLATURE

E_m = Modulus of elasticity of the matrix, N/m^2
E_f^i = Modulus of elasticity of the i-th real fibre, N/m^2
E_l^i = Modulus of elasticity of lateral fibres associated with i-th real fibre, N/m^2
n = Number of fibres that are oriented at different angles in a lamina
p_i = Fraction of total fibres lying in a band at angle ϕ_i to the principal direction
V_f = Fibre volume fraction
V_v = Fraction of the total volume taken by voids and cracks
ε_x = Normal strain in the principal direction
ε_y = Normal strain in the direction perpendicular to principal direction
ε_{xy} = Shear strain in xy plane
ϕ_i = Angle of between i-th band of real fibres and the principal direction
λ = Empirical constant for shear strength prediction of the composite
μ = Empirical constant allowing for the contiguity effects
ν = Poisson's ratio of the resin matrix
σ_x = Normal stress in the principal direction, N/m^2
σ_{xy} = Shear stress in xy plane, N/m^2
φ = Half the included angle between the lateral fibres

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APPENDIX (Bishop's Model)

Bishop has introduced the concept of lateral fibre which takes into account the indirect but substantial contributions of the real fibres to transverse and shear stiffnesses. These contributions are ignored by the netting analysis. While there are more sophisticated methods to compensate for these effects, the lateral fibre concept provides a computationally simple and convenient method. It is assumed that there exist two fictitious lateral fibres with an included angle of 2φ associated with each unidirectional real fibre. These lateral fibres compensate for the indirect contributions of real fibres to transverse and shear stiffnesses.

The basic model, with necessary modifications, is adopted for the present investigation. Details of the model is provided in [1]. The final form of the prediction equations can be stated as follows:

$$\begin{aligned}\sigma_x &= (\alpha_1)_T\,\varepsilon_x + (\beta_1)_T\,\varepsilon_y + (\gamma_1)_T\,\varepsilon_{xy}\\ \sigma_y &= (\alpha_2)_T\,\varepsilon_x + (\beta_2)_T\,\varepsilon_y + (\gamma_2)_T\,\varepsilon_{xy}\\ \sigma_{xy} &= (\alpha_3)_T\,\varepsilon_x + (\beta_3)_T\,\varepsilon_y + (\gamma_3)_T\,\varepsilon_{xy}\end{aligned} \qquad (A.1)$$

where the stiffness coefficients in Eq. (A.1) are symmetric and are given by:

$$\begin{aligned}(\alpha_1)_T &= \frac{1}{8}V_f\,(3C_o + 4C_2 + C_4) + \frac{(1\text{-}V_f\text{-}V_v)E_m}{(1\text{-}\nu^2)}\\ (\beta_1)_T &= (\alpha_2)\,\frac{1}{8}V_f\,(3C_o - C_4) + \frac{\nu(1\text{-}V_f\text{-}V_v)E_m}{(1\text{-}\nu^2)}\\ (\beta_2)_T &= \frac{1}{8}V_f\,(3C_o - C_2 + C_4) + \frac{(1\text{-}V_f\text{-}V_v)E_m}{(1\text{-}\nu^2)}\end{aligned} \qquad (A.2)$$

$$\begin{aligned}(\gamma_1)_T &= (\alpha_3)_T = \frac{1}{8}V_f\,(2S_2 + S_4)\\ (\gamma_2)_T &= (\beta_3)_T = \frac{1}{8}V_f\,(2S_2 - S_4)\\ (\gamma_3)_T &= V_f\,(C_o - C_4) + \frac{\lambda(1\text{-}V_f\text{-}V\,)E_m}{(1+\nu)}\end{aligned} \qquad (A.3)$$

The values of C_o, C_2, C_4, S_2, and S_4 in Eq. (A.2) are obtained from the following equations:

$$\begin{aligned}C_0 &= \sum_{i=1}^{n} p_i\,[\,E_f^i + 2E_1^i\,(1+\mu V_f)\\ C_2 &= \sum_{i=1}^{n} p_i \cos 2\phi_i\,[\,E_f^i - 2E_1^i\,(1+\mu V_f)\cos 2\Psi]\\ C_4 &= \sum_{i=1}^{n} p_i \cos 4\phi_i\,[\,E_f^i + 2E_1^i\,(1+\mu V_f)\cos 4\Psi]\\ C_2 &= \sum_{i=1}^{n} p_i \sin 2\phi_i\,[\,E_f^i - 2E_1^i\,(1+\mu V_f)\sin 2\Psi]\\ C_4 &= \sum_{i=1}^{n} p_i \sin 4\phi_i\,[\,E_f^i + 2E_1^i\,(1+\mu V_f)\sin 4\Psi]\end{aligned} \qquad (A.4)$$

It should be noted that the empirical parameters values given in section 3 can be used in Eqs. (A.2) and (A.3) to obtain the stiffness and strength properties of the composite laminates.