Note on Secondary Flow Vortex for Laminar Flow Through Circular Curved Pipes

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Abstract : This paper presents a numerical investigation of secondary flow development for laminar flow through circular curved pipes at two Dean numbers. The development of the secondary flow vortex is analysed in the form of streaklines. The trajectory of the vortex centres show that soon after their formation near the outside radius of the bend, the vortices migrate towards the inner radius very rapidly. Further readjustment of secondary flow vortex is observed accompanied by a monotonic "turning back" of the vortex centre. A weak separation of the secondary flow is predicted near the inner radius for the higher Dean number flow.

Keywords : *Vortex, Curved pipes, Laminar flow.*

INTRODUCTION

When a flow enters into a curved pipe, a centripetal acceleration is experienced, driven by an inwardly acting radial pressure gradient. The magnitude of this gradient is set by the bulk fluid velocity in the core of the flow. It thus represents an excessive force, out of balance with the local acceleration, for the slower moving fluid in the boundary layers on the walls of the pipe. These slower moving streams are caused to move inwards and, by continuity, outward flow is induced in the pipe centre. This curvature-induced secondary flow (significant transverse velocity over the cross-section), when superimposed on the stream wise flow gives rise to counter-rotating helical secondary-flow vortices as shown in Fig. 1. It is obvious that the flow in the entrance region of curved pipes is fully three-dimensional. Theoretical attempts to solve flows in the entrance region are seriously hindered by several (extremely) simplifying assumptions [1,2,3].

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On the other hand, the experimental works on developing flow in circular sectioned curved pipes are also limited because of experimental limitations. The situation is further complicated by the small magnitude of secondary velocity components such as the ones occurring in separating/near-separating conditions [4,5]. The stream wise velocities, however , can be measured more accurately and the development pattern of this velocity component is known to a greater extent. As pointed out earlier that since the flow in the developing region is fully three-dimensional, the understanding of the more established stream wise velocity cannot be complete without a detailed knowledge of the secondary flow.

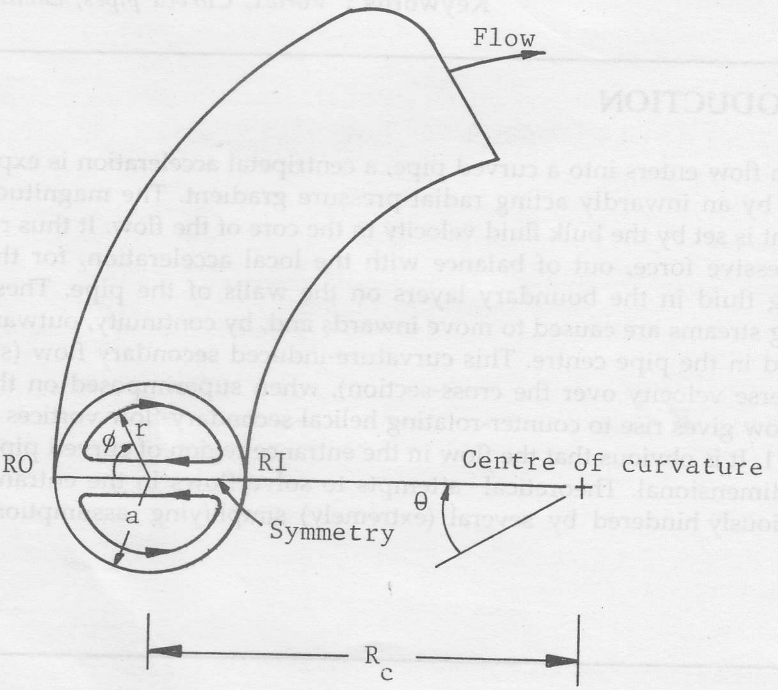

Fig. 1 Coordinate system and typical secondary flow in a curved pipe.

During the last two decades several researchers have reported the results of numerical work on this type of flow. Of them, the numerical works of Soh and Berger [6] and Humphrey et al [7] appear to be the most detailed. They have numerically solved the flow through curved pipes for different De and δ. De represents the Dean number defined as $De = Re\ \delta^{1/2}$ where Re is the Reynolds number and δ is the radius ratio, defined as a/Rc (Fig.1). In these two papers, the development of secondary flow has been analysed by considering the velocity vectors at successive stream wise cross-sectional planes. A secondary flow separation near the inner radius is also reported. However, the fact that the development of secondary flow is a two-fold phenomenon (unlike that of the stream wise velocity) has not been given attention apart from a qualitative comment on this issue. In this paper, this aspect is given prior importance by considering the secondary-flow development in the form of streaklines. Results are presented for two Dean numbers (De = 183 , δ = 1/7 and De = 566, δ = 1/20) . Circumferential shear stress is also drawn for one of the cases in order to show the separation of secondary flow.

SOLUTION PROCEDURE AND BOUNDARY CONDITIONS

The flow is analysed in toroidal r, θ, ϕ system shown in Fig. 1. Here, θ is the streamwise coordinate and ϕ, r are the cross-stream coordinates. R_c and 'a' represent the radius of curvature and radius of the pipe cross-section respectively. The equations of motion for a general variable ψ can be written as :

$$\frac{1}{rr_c}\left[\frac{\partial}{\partial r}(\rho r r_c\, v\psi) + \frac{\partial}{\partial \phi}(\rho r_c\, w\psi) + \frac{\partial}{\partial \theta}(\rho r u\psi)\right] + S_C(\psi)$$

$$= S_P(\psi) + \frac{1}{rr_c}\left[\frac{\partial}{\partial r}\{\mu r r_c \frac{\partial \psi}{\partial r}\} + \frac{\partial}{\partial \phi}\{\mu \frac{r_c}{r}\frac{\partial \psi}{\partial \phi}\} + \frac{\partial}{\partial \theta}\{\mu \frac{r}{r_c}\frac{\partial \psi}{\partial \theta}\}\right] + S_D(\psi) \quad \text{......} \quad (1)$$

where $r_c = R_c + r\cos\phi$ and v,u,w are the three velocity components in r, θ and ϕ direction respectively. The expressions for S_P, S_C and S_D for different equations are shown in Table 1.

Table 1 Source Terms for Different Equations

Equation	Value of (ψ)	S_P (ψ)	$S_C(\psi)$	$S_D(\psi)$
Continuity Equation	1	0	0	0
u-momentum Equation	u	$-\frac{1}{r_c}\frac{\partial p}{\partial \theta}$	$\frac{1}{r}\rho vu\cos\phi$ $-\frac{1}{r_c}\rho wu\sin\phi$	$\frac{2\mu}{r_c^2}\left(\frac{\partial v}{\partial \theta}\cos\phi - \frac{\partial w}{\partial \theta}\sin\phi - \frac{u}{2}\right)$
v-momentum Equation	v	$\frac{\partial p}{\partial r}$	$\frac{rw^2}{r} - \frac{ru^2\cos\phi}{r}$	$\frac{\mu}{r^2}\left(2\frac{\partial w}{\partial \phi} + v\right) + \frac{\mu w}{rr_c}\sin\phi$ $+\frac{\mu\cos\phi}{r_c^2}\left(w\sin\phi - v\cos\phi - 2\frac{\partial u}{\partial \theta}\right)$
w-momentum Equation	w	$-\frac{1}{r}\frac{\partial p}{\partial \phi}$	$\frac{1}{r}\rho vw + \frac{1}{r_c}\rho u^2 \sin\phi$	$\frac{\mu}{r^2}\left(2\frac{\partial v}{\partial \phi} - w\right) - \frac{\mu}{rr_c}v\sin\phi$ $-\frac{\mu}{r_c^2}\sin\phi\left(w\sin\phi - v\cos\phi - 2\frac{\partial u}{\partial \theta}\right)$

In order to obtain a numerical solution, the equations given by Eq.1 are first discretised and converted to algebraic equations [8,9]. During the process of discretisation some assumptions and approximations have been made following Patankar [8]. The resulting computer code has been found to give correct result for known test cases such as developing laminar flows through straight pipe and through two-dimensional curved duct. A non-uniform 20 x 30 x 20 (r x θ x ϕ) grid distribution is used for the final calculation. This particular mesh is found to yield almost grid independent results [9]. The convection scheme has been taken to be the 'hybrid' differencing scheme [8] and the solution algorithm has been taken to be SIMPLE.

The boundary conditions at the inlet are taken to be inviscid free vortex type for the stream wise component following the experimental observations by Agrawal et al [4]. The cross-stream components of velocity (v, w) are set to be zero. At the exit (θ=180 deg), a zero-gradient boundary condition is prescribed. It should be noted that the zero-gradient boundary condition is not strictly valid for curved pipes. However, the results presented in this paper are upto θ=140 degree; and it has been checked that this particular boundary condition at the exit does not affect the results by more than 20-deg upstream. Finally, the no-slip condition is chosen for the walls and symmetry conditions are imposed on the plane of symmetry. Since all the r-lines originate from the axis of the pipe at every cross-section, a singularity problem arises there. This has been tackled in a fashion as described in [7]. Calculations are done for the top-half of pipe for two Dean numbers of De = 183 ($\delta = 1/7$) and De = 566 ($\delta = 1/20$).

RESULTS AND DISCUSSION

The development of secondary flow for the two test cases are presented in Figs. 2 and 3 in the form of streaklines (analogous to a flow visualization). Just after the inlet (θ = 3 deg; Figs. 2a and 3a), a secondary velocity acts radially inward. This is due to the displacement effect of the boundary layers as the flow moves into the bend. Soon after this, the secondary velocities near the wall become almost parallel to the curved surface with a gradual development of the circumferential boundary layer. The secondary velocities near the outer wall (RO) change direction and give rise to a secondary-flow vortex. A closer observation reveals that the centre of this secondary flow vortex change position from near the outer bend towards the inner (i.e., increasing ϕ) as the flow moves downstream. The paths of movement of the vortex centres are shown in Fig. 4. It can be seen that for lower value of De, the vortex motion is established at higher values of θ. Interestingly, after reaching the rightmost positions in the cross-section (i.e., θ=38 and 65 -deg for De=183 and 566 respectively), the secondary flow readjusts itself and monotonically "turns back" in the opposite direction and finally reaches the fully-developed state. ** The angle through which this "turning back" takes place is 12 deg and 5 deg for De = 183 and 566 respectively. It is also seen that the circumferential boundary layer is squeezed more towards the curved wall for higher De. The reason for this "turning back" can be explained as follows. In the early stages of flow, the secondary flow develops very rapidly with little interaction with the mean stream wise flow and hence the secondary-flow vortex reaches its maximum inside position (rightmost position on the cross-section) quickly, whereas in the later stage, the centrifugal forces become more dominant and hence drags the secondary-flow vortex in the opposite direction.

*** Strictly speaking the flow in a curved pipe changes continuously even if the exit is at a very large value of θ. However, the term fully-developed is used by many authors [5,6,7] and in this work the flow is declared to reach fully-developed situation when the change in secondary velocity component (v, w) is less than 0.5% between successive stream wise cross-sectional planes.*

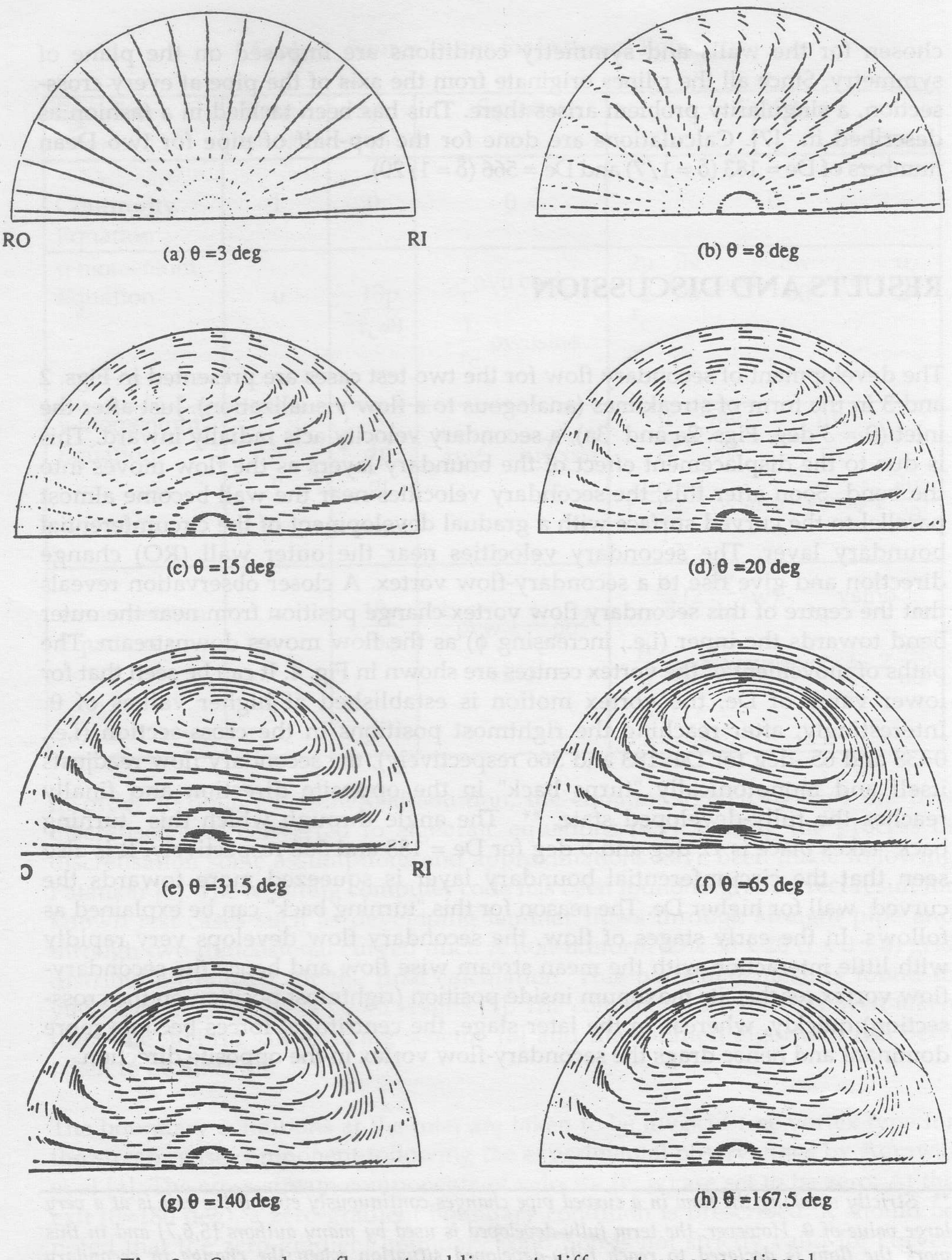

Fig. 2 Secondary flow streaklines at different cross-sectional locations of curved pipe, (De=183).

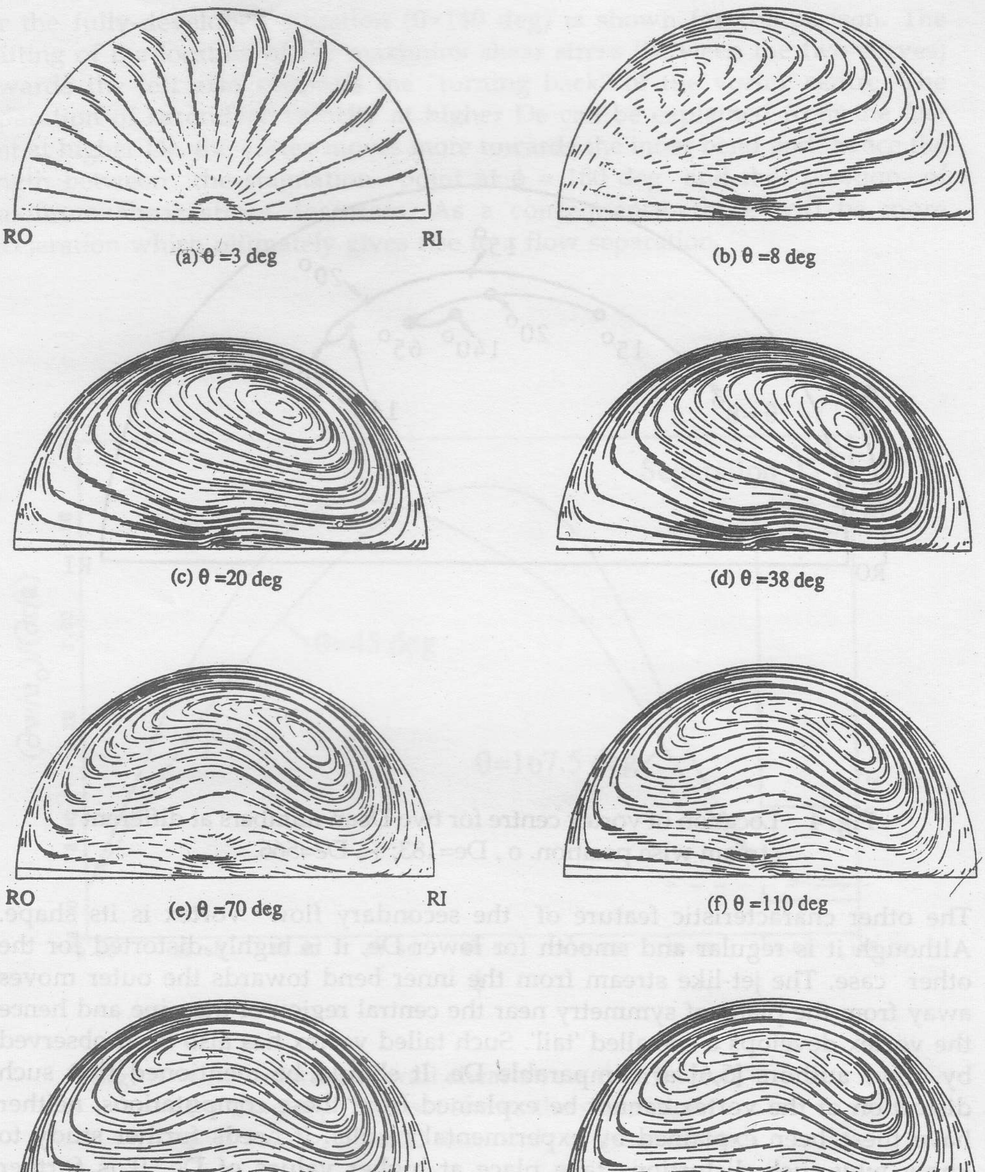

Fig. 3 Secondary flow streaklines at different cross-sectional locations of curved pipe (De=566).

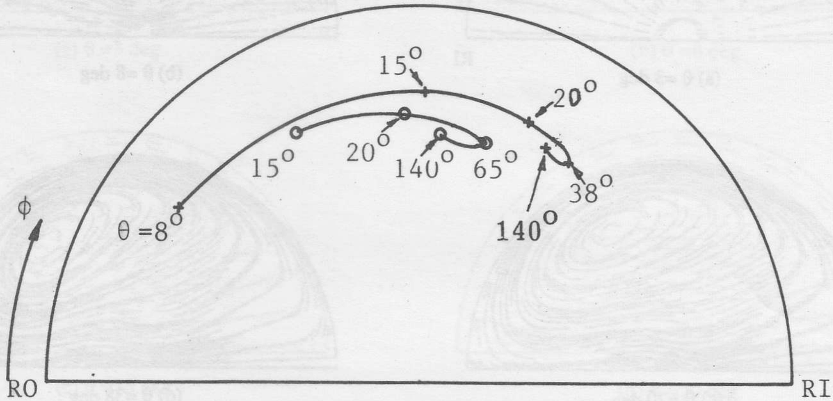

Fig. 4 Location of vortex centre for two Dean numbers at different stream wise position. o , De=183; + , De=566.

The other characteristic feature of the secondary flow vortex is its shape. Although it is regular and smooth for lower De, it is highly distorted for the other case. The jet-like stream from the inner bend towards the outer moves away from the plane of symmetry near the central region of the pipe and hence the vortex develops a so-called 'tail'. Such tailed vortex has also been observed by other authors [5,6] at comparable De. It should be mentioned that such distortion of the vortex cannot be explained from these computations, neither have these been explained by experimental results. It needs further study to know why such distortions take place at higher values of De. It is further revealed that a weak separation of the circumferential boundary layer is present for De = 566 between stream wise locations of θ = 38 deg and 52 deg. The existence of this separation can be seen from the circumferential shear stress plots in Fig. 5. A magnified view (x20, only along the ordinate) of this separation region is shown in the inset. The separation bubble is located within the last 10 deg of the cross-section. The shear stress (calculated by dividing the w-component on the grid node adjacent to the curved wall by the distance of that grid from the wall) for the fully-developed situation (θ=140 deg) is shown for comparison. The shifting of the location of the maximum shear stress (between the two curves) towards the left also supports the "turning back" of the vortex centre. The separation of secondary velocity at higher De can be explained from the fact that at higher De, the vortex moves more towards the inner bend and hence the length between the stagnation point at ϕ = 180 deg and the position of maximum shear stress decreases. As a consequence, there will be more deceleration which ultimately gives rise to a flow separation.

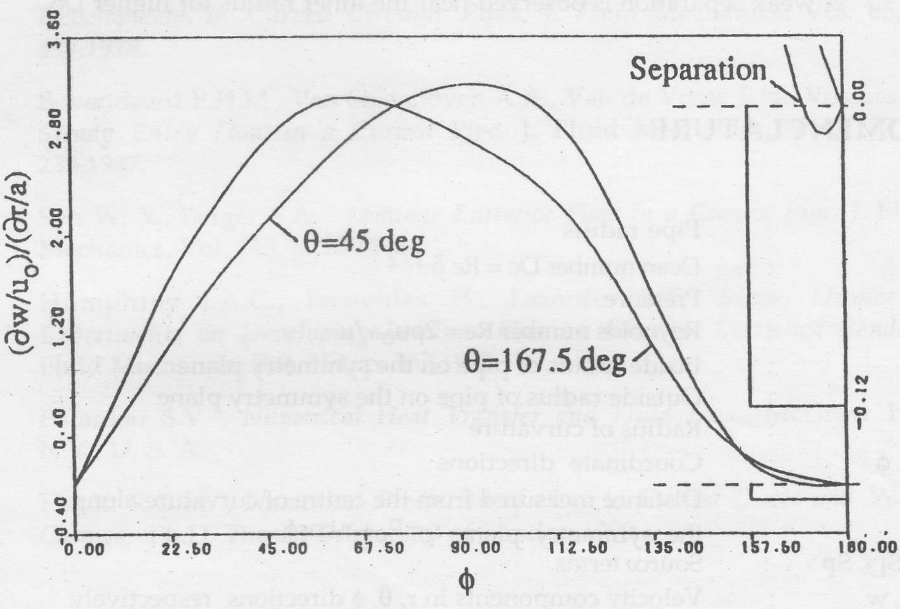

Fig. 5 Circumferential wall shear stress distribution for De=566. The inset shows the separation region magnified 20 times only in the ordinate.

CONCLUSIONS

1. The development of secondary flow vortex is a two-fold phenomenon. The first part is a rapid migration of vortex centre from near the outer radius of bend towards the inner and the second is a monotonic "turning back" in the opposite direction.

2. With larger De, the vortex moves further towards the inner radius and away from the plane of symmetry.

3. A weak separation is observed near the inner radius for higher De.

NOMENCLATURE

a	:	Pipe radius
De	:	Dean number $De = Re\,\delta^{1/2}$
p	:	Pressure
Re	:	Reynolds number $Re = 2\rho u_o a/\mu$
RI	:	Inside radius of pipe on the symmetry plane
RO	:	Outside radius of pipe on the symmetry plane
R_c	:	Radius of curvature
r, θ, ϕ	:	Coordinate directions
r_c	:	Distance measured from the centre of curvature along the symmetry plane; $r_c = R_c + r\cos\phi$
S_C, S_D, S_P	:	Source terms
v, u, w	:	Velocity components in r, θ, ϕ directions respectively
u_o	:	Bulk velocity
δ	:	Radius ratio $\delta = a/R_c$
ρ	:	Density of fluid
μ	:	Dynamic viscosity of fluid
ψ	:	General variable

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