Mech. Engg. Res. Bull.
Vol. 14 (1991), pp 35-46

# Finite Difference Solution of Wakes for Turbulent Flow Behind a Flat Plate

**Md. Tazul Islam***
**Dr. S. M. Nazrul Islam****

## ABSTRACT

*The present investigation was on the meanflow parameters of the wake, identifying the initial conditions and a suitable turbulent shear stress model. The equations for mass and momentum conservation for turbulent flow were solved numerically by using appropriate boundary conditions and Finite Difference Scheme. Mean properties of the flow at the trailing edge were expressed by empirical relation using on experimental values of Faruque [1983].*

*Prandtl's mixing length was expressed as a function of shear layer thickness and it was used in the turbulent shear stress model.*

*The calculated results of the different flow parameters of the wake show agreement with experimental results.*

*And also shows that Prandtl's mixing length model is satisfactory to express the turbulent shear stress in the wake.*

## NOMENCLATURE

| | |
|---|---|
| u,v | Axial & transverse mean velocity |
| x,y | Co-ordinate system |
| $\rho$ | Mean density |
| $\gamma$ | Eddy diffusivity |
| u',v' | Fluctuating velocities in x & y direction |
| $\tau$ | Shear stress |
| l | Prandtl's mixing length |
| $\chi_i$ | Constant used to define prandtl's new formula |
| $\delta$ | Boundary layer thickness |
| n | Power Index |
| $u_\infty$ | Free-stream velocity |
| D | Plate thickness |
| $Y_{1/2}$ | Half width |
| $C_{Dm}$ | Drag Co-efficient (without pressure gradient) |
| L | Characteristic length |
| H | Shape factor |
| $\delta^*$ | Displacement thickness |
| $Re\theta e$ | Reynolds number based on the momentum thickness & average velocity |
| $\theta$ | Momentum thickness |
| rms | Root mean square |
| $u_c$ | Centre line velocity |
| U & V | non-dimensional mean velocity $u/u_\infty$, $v/u_\infty$ |
| X & Y | non-dimensional axial & transverse distances |

## INTRODUCTION

A wake is formed behind a solid body which is being towed through a fluid at rest or behind a solid body which has been immersed in a stream of a fluid. Fig.1. An exchange of momentum takes place continuously in the wake from the high velocity region to the low velocity region. Typically high level of momentum exchange together with a change of fluid properties make the flow to be turbulent. The wake generated in such turbulent flow is said to be turbulent wake.

The characteristic features of turbulent wake are important for many practical engineering application. A maneuvering air craft or submarine which is accelerating or decelerating leaves behind it a momentum defect in the form of jet or wake when it changes speed. All such wakes are turbulent in character and these need to be studied for drag and other parameters.

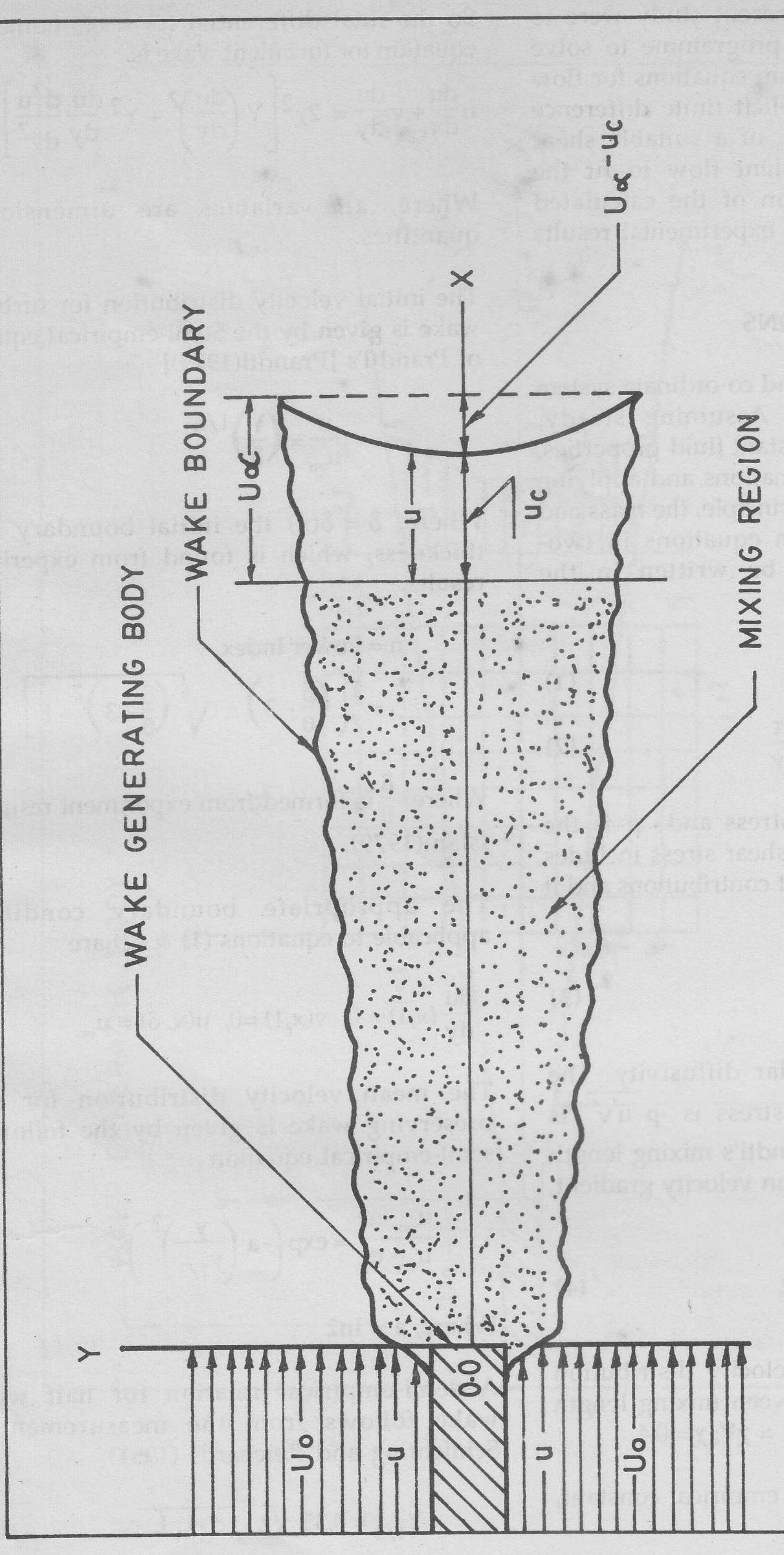


FIG 1 WAKE GEOMETRY AND NOMENCLATURE

The objectives of the present study were to developed a computer programme to solve continuity and momentum equations for flow properties using an explicit finite difference technique. Identification of a suitable shear stress model for turbulent flow to fit the solution and comparison of the calculated results with the existing experimental results of Faruque (1983).

## GOVERNING EQUATIONS

The flow configuration and co-ordinate system are shown in Fig. 2. Assuming steady, incompressible flow, constant fluid properties, Boundary layer approximations and applying the order of magnitude principle, the mass and momentum conservation equations in two-dimensional flow can be written in the following form.

$$\frac{du}{dx}+\frac{dv}{dy}=0 \qquad (1)$$

$$u\frac{du}{dx}+v\frac{du}{dy}=\frac{1}{\rho}\frac{d\tau}{dy} \qquad (2)$$

Where, $\tau$ is the shear stress and $\rho$ is the density of the fluid. The shear stress includes both viscous and turbulent contributions and it is written as

$$\frac{\tau}{\rho}=\gamma\frac{du}{dy}-\overline{u'v'} \qquad (3)$$

Where, $\gamma$ is the molecular diffusivity. The turbulent part of shear stress is $-\rho\,\overline{u'v'}$ is expressed in terms of Prandtl's mixing length [Prandtl (1925)], l and mean velocity gradient, du/dy

$$\frac{\tau}{\rho}=\gamma\frac{du}{dy}-l^2\left(\frac{du}{dy}\right)^2 \qquad (4)$$

According to prandtl's velocity distribution principle the relation between mixing length and vertical distance, Y is $l = \chi Y$; $\chi = 0.4$.

Where, $\chi$ = dimensionless empirical constant, obtained from experiments.

Hence, according to prandtl's assumption, by neglecting molecular diffusivity term the turbulent shear stress becomes.

$$\tau=\chi^2\rho Y^2\left(\frac{du}{dy}\right)^2 \qquad (5)$$

So the final differential form of momentum equation for turbulent wake is

$$u\frac{du}{dx}+v\frac{du}{dy}=2\chi^2\left[Y\left(\frac{du}{dy}\right)^2+Y^2\frac{du}{dy}\frac{d^2u}{dy^2}\right] \qquad (6)$$

Where, all variables are dimensionless quantities.

The initial velocity distribution for turbulent wake is given by the Semi-empirical equation of Prandtl's [Prandtl(1925)]

$$\frac{u}{u_\infty}=\left(\frac{Y}{\delta}\right)^{1/n} \qquad (7)$$

Where, $\delta = \delta(x)$ the initial boundary layer thickness, which is found from experiment results.

n = Power Index.

$$=\frac{1}{2}\left[\left(\frac{\delta}{\theta}-3\right)\pm\sqrt{\left(\frac{\delta}{\theta}-3\right)^2-8}\right] \qquad (8)$$

Where, $\frac{\delta}{\theta}$ is formed from experiment results of Islam(1979).

The appropriate boundary conditions applicable to equations (1) & (2) are

$$\frac{du}{dy}(x,1)=0,\ v(x,1)=0,\ u(x,\delta)=u_\infty \qquad (9)$$

The mean velocity distribution for self-preserving wake is given by the following semi-empirical equation

$$\frac{u_\infty-u}{u_\infty-u_c}=\exp\left(-a\left(\frac{y}{y_{1/2}}\right)^2\right) \qquad (10)$$

Where, a = ln2

A semi-empirical relation for half width wake follows from the measurement of Schlichting and Reichardt (1951)

$$Y_{1/2}=0.35\sqrt{x}\sqrt{c_{dm}L} \qquad (11)$$

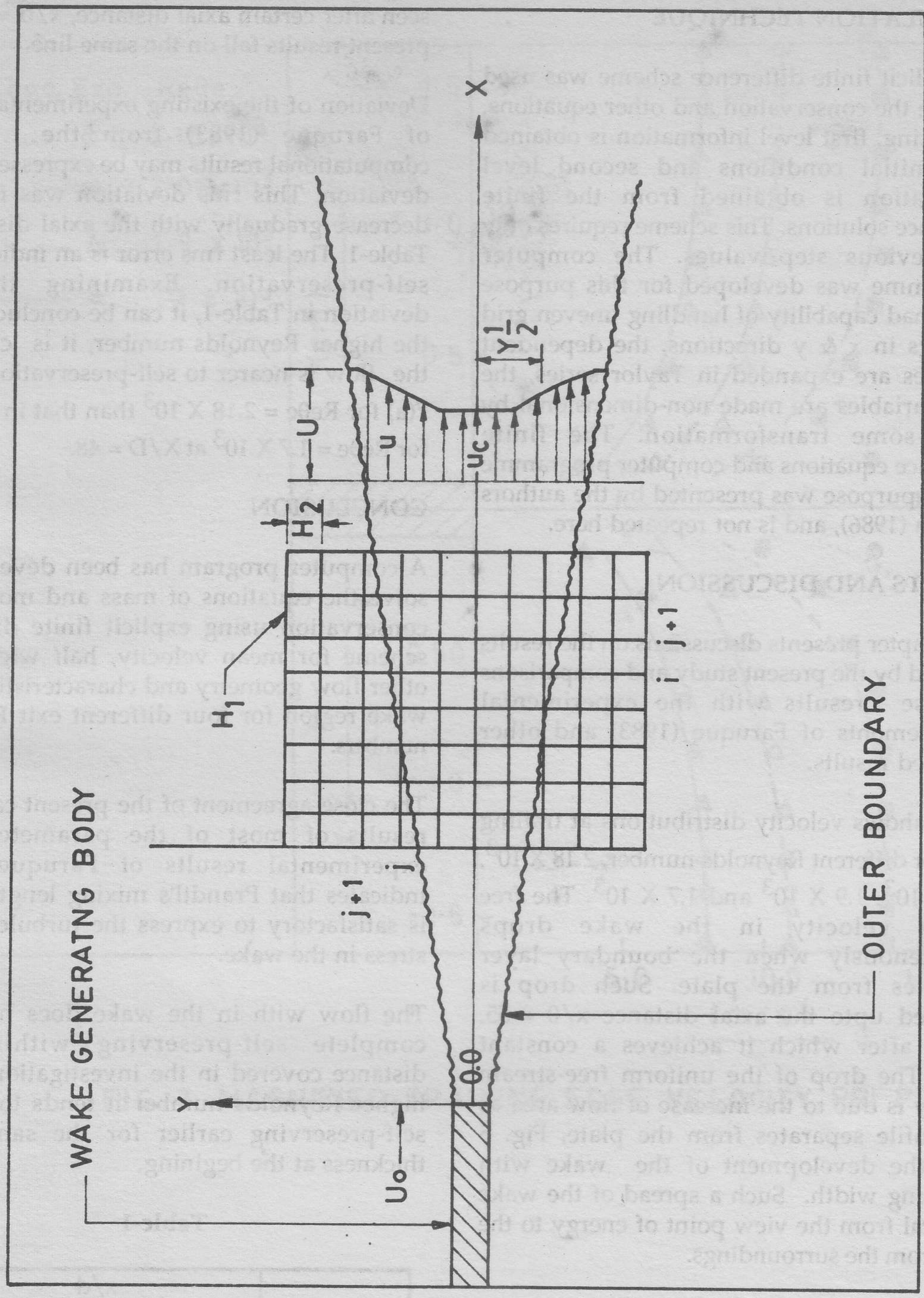


FIG. 2 CO-ORDINATE SYSTEM OF WAKE AND FINITE-DIFFERENCE GRIDS

## CALCULATION TECHNIQUE

An explicit finite difference scheme was used to solve the conservation and other equations. At starting, first level information is obtained from initial conditions and second level information is obtained from the finite difference solutions. This scheme requires only the previous step values. The computer programme was developed for this purpose which had capability of handling uneven grid spacings in x & y directions, the dependent variables are expanded in Taylor series, the basic variables are made non-dimensional by using some transformation. The finite difference equations and computer programme for this purpose was presented by the authors in Islam (1986), and is not repeated here.

## RESULTS AND DISCUSSION

This chapter presents discussions on the results obtained by the present study and comparisons of these results with the experimental measurements of Faruque (1983) and other predicted results.

Fig. 3. shows velocity distributions at trailing edge for different Reynolds number, $2.18 \times 10^3$, $2.01 \times 10^3$, $1.9 \times 10^3$ and $1.7 \times 10^3$. The free stream velocity in the wake drops instantenously when the boundary layer separates from the plate. Such drop is observed upto the axial distance $x/\theta = 55$, (Fig.4) after which it achieves a constant value. The drop of the uniform free-stream velocity is due to the increase of flow area as the profile separates from the plate. Fig. 5 show the development of the wake with increasing width. Such a spread of the wake is logical from the view point of energy to the wake from the surroundings.

The shape factor, $H = \delta^*/\theta$, is plotted in Fig. 6 as a function of axial distance. The slope of the curve for any Reynolds number decreases at a higher rate in the region close to the trailing edge and at a slower rate with the increase of axial distance. This trend of the shape factor curves in Fig. 6 shows an indication of self-preservation of flow. The flow will be absolutely self-preserving when the shape factor tends to be unity. Dimensionless velocity distribution for the wakes are shown in Fig. 7(a), Fig. 7(b), to examine their self-preservation. The half width $Y_{1/2}$ is used a length scale in the self- preservation plot. It is seen after certain axial distance, $x/d = 16$. The present results fall on the same line.

Deviation of the existing experimental results of Faruque (1983) from the present computational results may be expressed as rms deviation. This rms deviation was found to decrease gradually with the axial distance in Table-1. The least rms error is an indication of self-preservation. Examining the rms deviation in Table-1, it can be concluded with the higher Reynolds number, it is clear that the flow is nearer to self-preservation in Fig. 7(a) for $Re\theta e = 2.18 \times 10^3$ than that in Fig. 7(b) for $Re\theta e = 1.7 \times 10^3$ at $X/D = 48$.

## CONCLUSION

A computer program has been developed to solve the equations of mass and momentum conservation using explicit finite difference scheme for mean velocity, half width and other flow geometry and characteristics in the wake region for four different exit Reynolds numbers.

The close agreement of the present calculated results of most of the parameters with experimental results of Faruque (1983) indicates that Prandtl's mixing length model is satisfactory to express the turbulent shear stress in the wake.

The flow with in the wake does not show complete self-preserving within axial distance covered in the investigation. But at higher Reynolds number it tends to become self-preserving earlier for the same plate thickness at the begining.

**Table-1**

| $Re\theta e$ | x/d | | |
|---|---|---|---|
| | 16 | 24 | 48 |
| $2.18 \times 10^3$ | 0.066 | 0.0620 | 0.0600 |
| $2.01 \times 10^3$ | 0.072 | 0.0690 | 0.0675 |
| $1.90 \times 10^3$ | 0.078 | 0.0760 | 0.0750 |
| $1.70 \times 10^3$ | 0.095 | 0.0935 | 0.0925 |

rms deviations at various distances from the trailing edge of the plate.

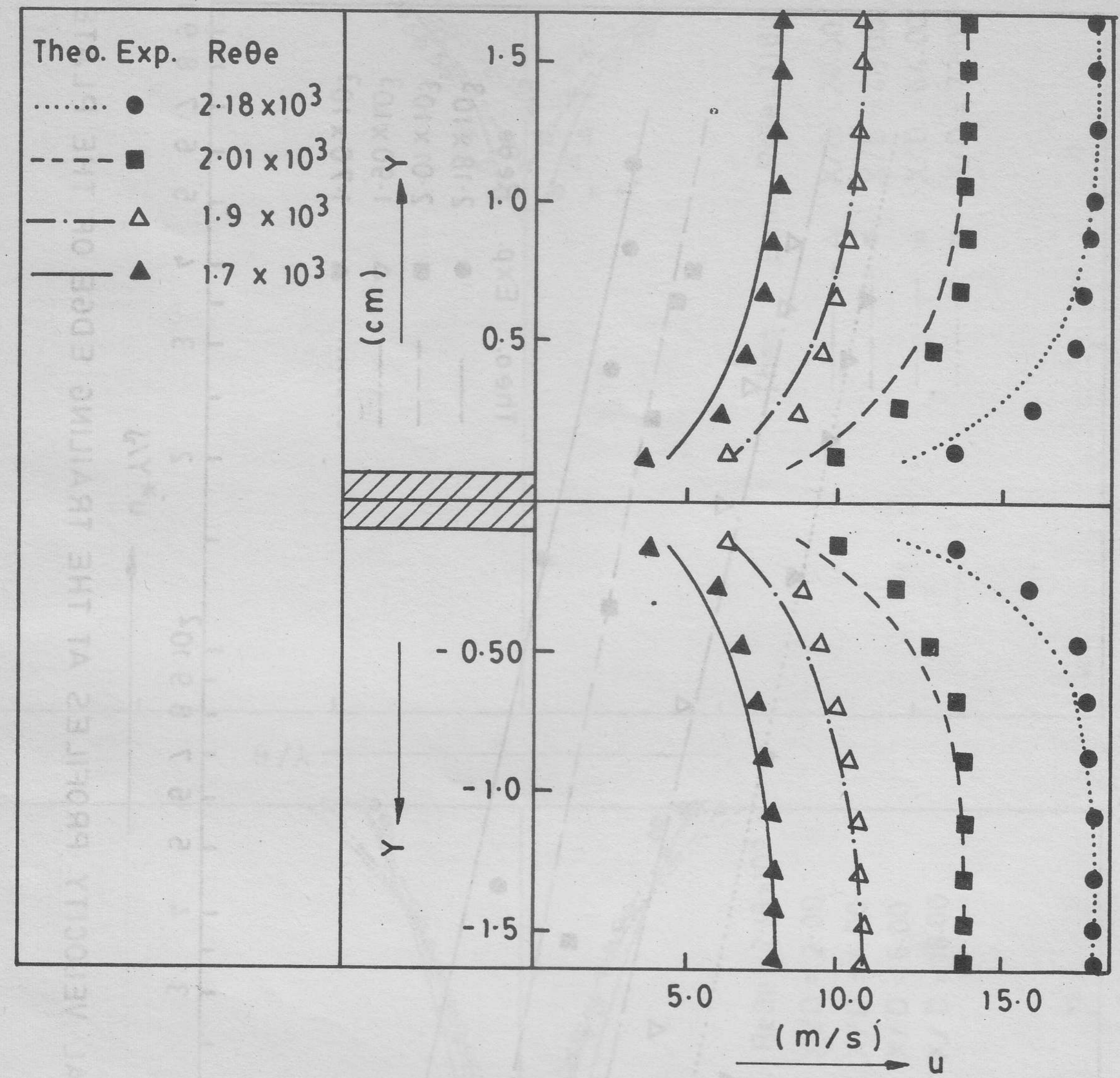


FIG. 3 MAGNIFIED TRAILING EDGE VELOCITY PRI PROFILE

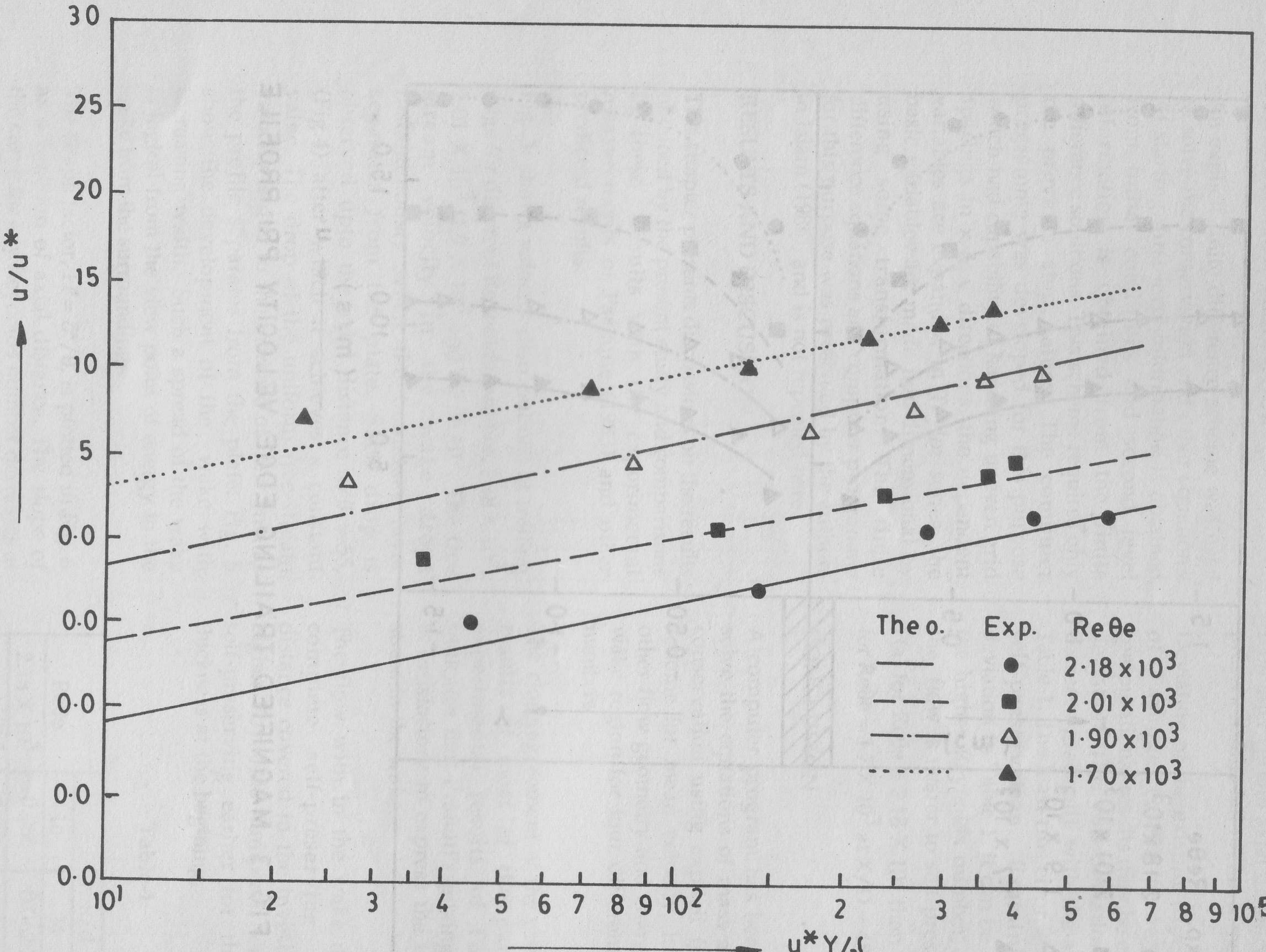


FIG. 4 UNIVERSAL VELOCITY PROFILES AT THE TRAILING EDGE OF THE PLATE

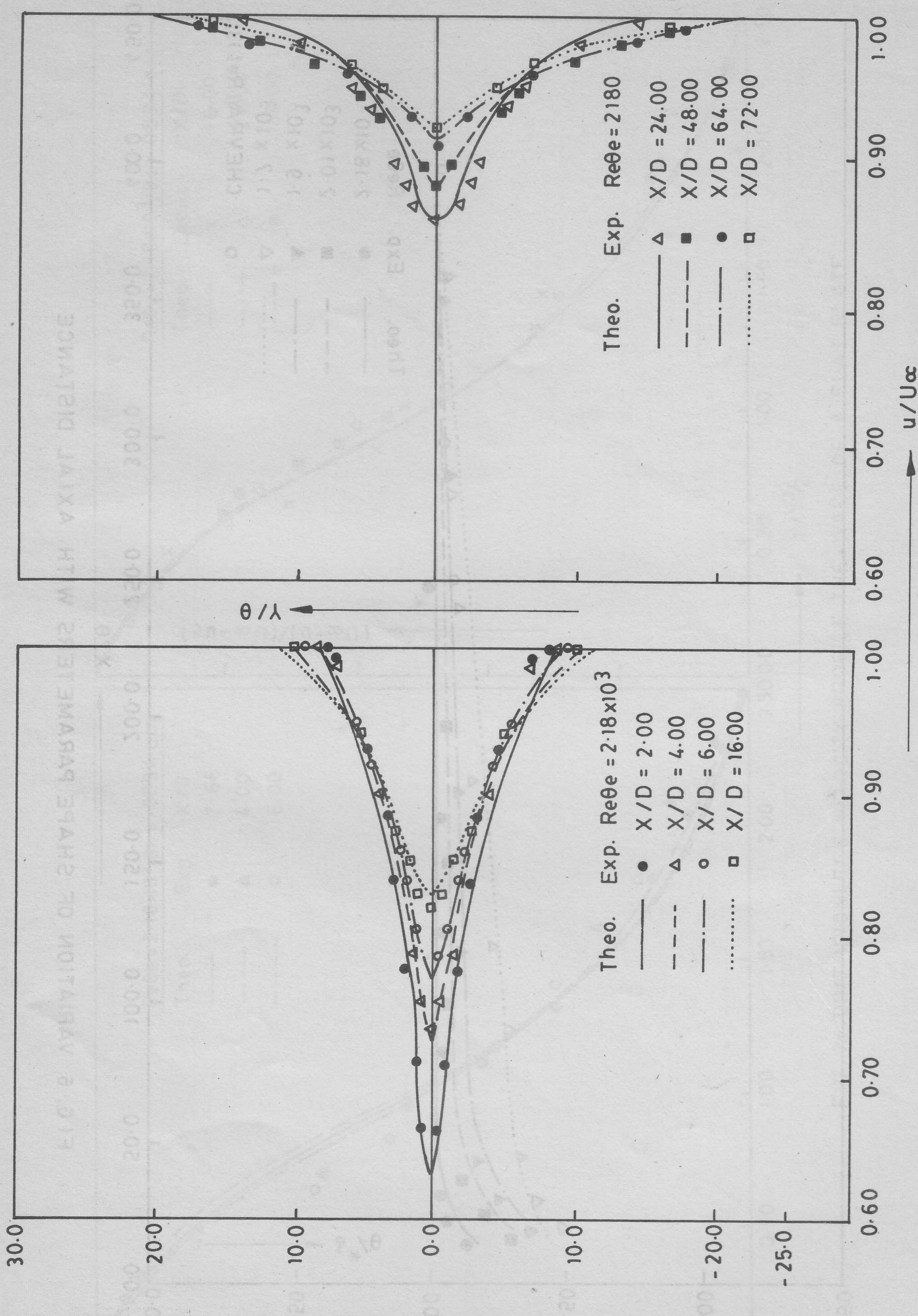


FIG. 5 MEAN VELOCITY DISTRIBUTION IN WAKE

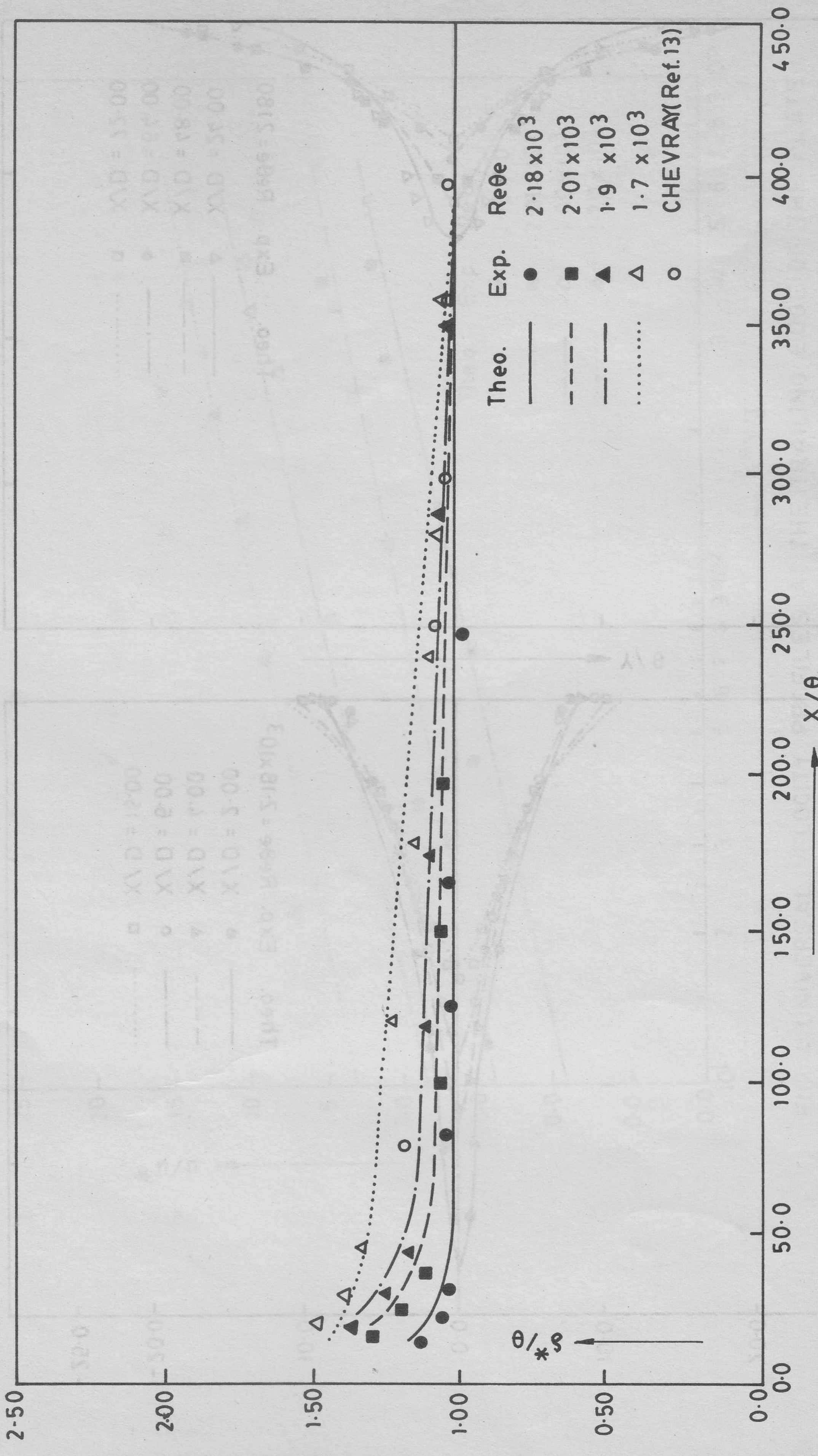


FIG. 6 VARIATION OF SHAPE PARAMETERS WITH AXIAL DISTANCE

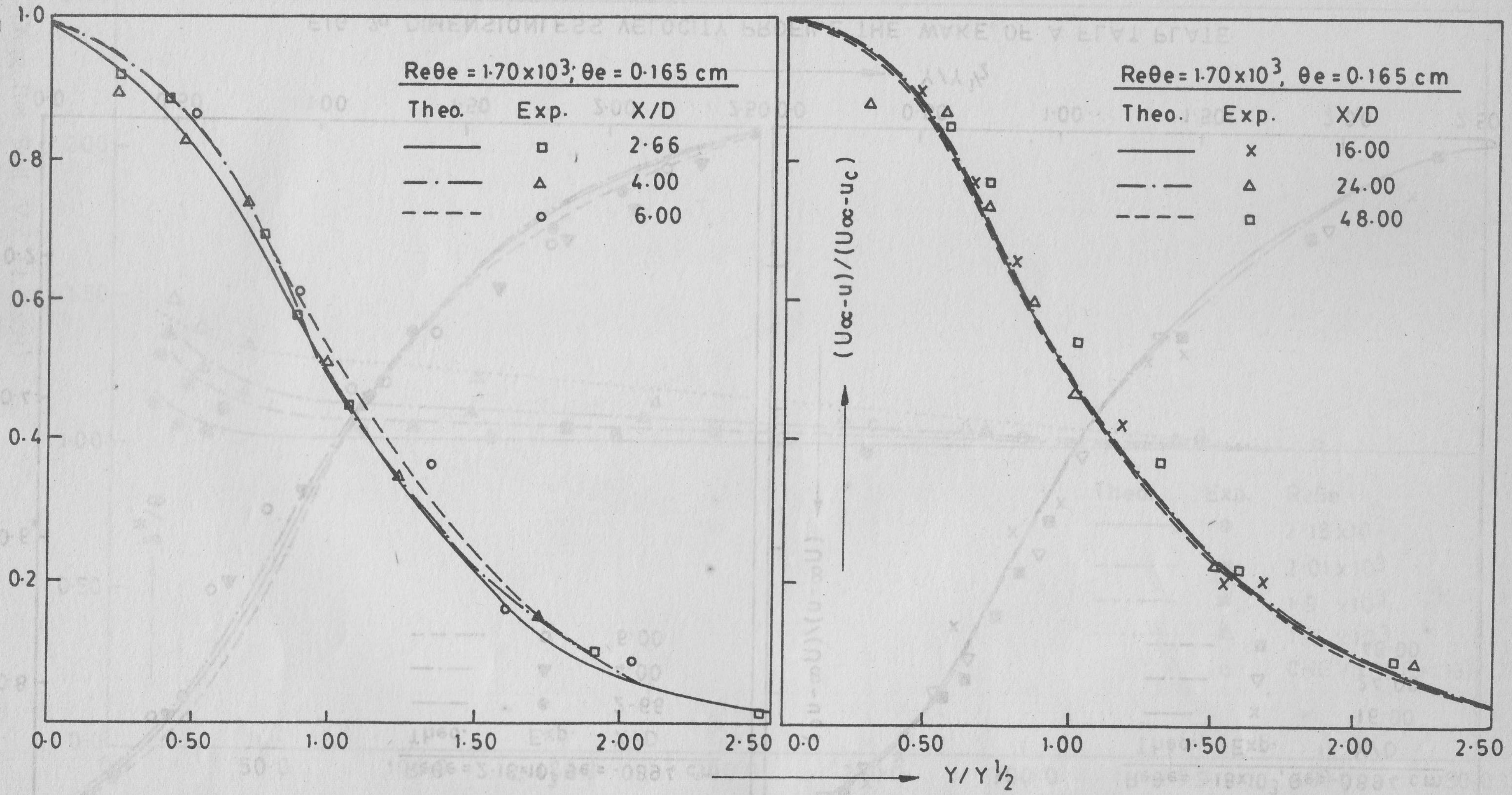


FIG. 7b DIMENSIONLESS VELOCITY PROFILE IN THE WAKE OF A FLAT PLATE

## REFERENCES


1. FARUQUE, P. K. O. (1983), "Experimental Investigation of Two Dimensional Wakes Behind Flat Plates", M.Sc. Engg. (Mechanical) Thesis. Dept. of Mechanical Engineering, BUET, Dhaka, Nov.

2. ISLAM, MD. TAZUL (1986), "Finite Difference Solution of Wakes for Turbulent Flow Behind a Flat Plate", M.Sc. Thesis, Department of Mechanical Engineering, BUET, Dhaka, Bangladesh, July.

3. ISLAM, S. M. NAZRUL (1979), "Prediction and Measurement of Turbulence in the Development Region of Axisymmetric Isothermal Jets", Ph.D. Thesis, Dept. of Mech. Engg., University of Windsor, Windsor, OWT, Canada.

4. ISLAM, S. M. NAZRUL (1975), "Design and Construction of Closed Circuit Wind Tunnel", M.Sc. Engg. Thesis, Dept. of Mechanical Engineering, BUET, Dhaka.

5. KEFEER, J. F. (1965), "The Uniform Distribution of a Turbulent Wake", J.F.M., Vol. 22, pp. 135.

6. KHALIL, G. M. (1982),"The Initial Region of Plane turbulent Mixing Layer", Ph.D. Thesis, Dept. of Mechanical Engineering, BUET, Dhaka.

7. PRANDTL, L. (1925), "Bericht Uber Untersuchungen Zur Ausgebildeten Turbulent", ZAMN, Vol. 5, pp. 136.

8. Reichardt, H. (1951), "Gesetzmaning Keiten Der Fkeien Turbulent", VDI-Forchungsh, pp. 414.

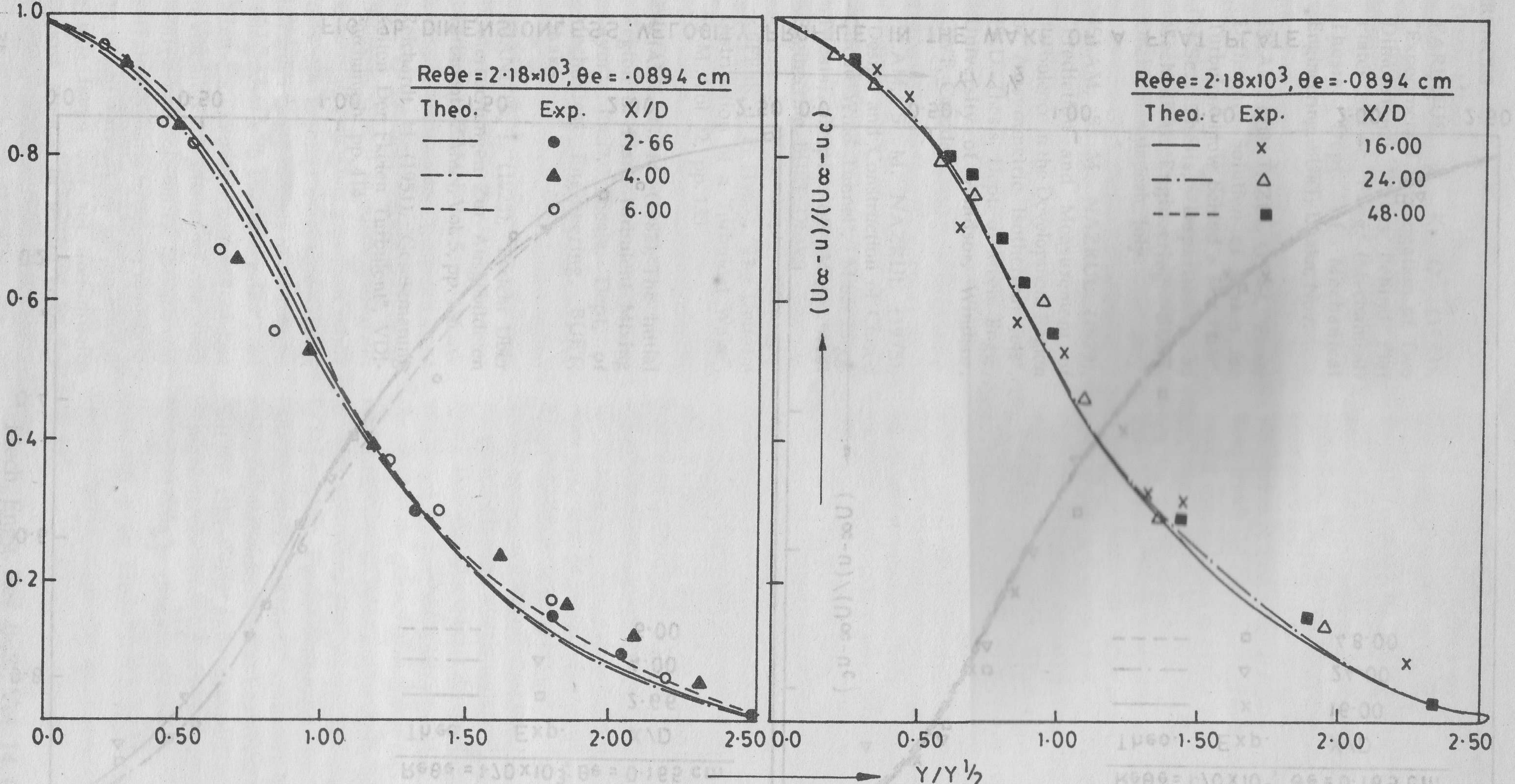


FIG. 7a DIMENSIONLESS VELOCITY PROFILE THE WAKE OF A FLAT PLATE