Mech. Engg. Res. Bull.
Vol. 12, (1989), pp. 37-54

Momentum and Kinetic Energy Correction Factors and Development of Turbulent Flow Through a Pipe

S. M. Nazrul Islam*
A. K. M. Sadrul Islam*

ABSTRACT

With a specified entrance cone the hydrodynamic flow development through a circular pipe is investigated for different Reynolds numbers. The flow in the pipe was turbulent and the developing length increased with the decrease of Reynolds number and Langhaar's relation was not applicable for this case. The momentum and energy correction factors increase in the developing region and attain constant maximum value in the developed region. These factors increase in the developing region and attain constant maximum value in the developed region. These factors depend upon Reynolds number and the empirical relationships for the factors are developed for the developing region.

A kiel probe and a static probe were used for measurements.

SYMBOLS

A	cross sectional area of the pipe
D	diameter of the pipe
L	developing length of flow
m	exponent of the empirical velocity eqn(1)
r	radial distance measured from pipe axis
r_o	radius of the pipe
R_e	Reynolds number based on V_a and diameter, V_aD/ν
RMS	root mean square
u	mean axial velocity of flow
u_c	central line mean axial velocity of flow

---

* Department of Mechanical Engineering, Bangladesh University of Engineering and Technology, Dhaka 1000.

V_a	area-average velocity of flow
x	axial distance measured from the inlet
y	distance measured from the wall of the pipe
α	kinetic energy correction factor
β	momentum correction factor
δ	boundary layer thickness
δ^*	displacement thickness of the profile
ρ	density of the fluid
θ	momentum thickness of the profile

INTRODUCTION

When a fluid enters a duct, the velocity profile develops gradually along the axial direction until it becomes fully developed at some downstream distances. The velocity profile in the developing region depends upon many factors, including the geometry at entrance and the surface roughness of the duct. Some scientists have worked on the developing region with laminar flow in ducts. Langhaar [1] developed an expression for the length of the developing region as a function of Reynolds number, $L/D = 0.058$ Re, for flow through pipes. This expression satisfies the experimental results for laminar flow in pipes with sharp entrance. But Langhaar's expression for the developing length does not show satisfactory agreement with the experimental results for different entrance geometry and surface for the laminar flow in the pipe. The flow in the duct becomes turbulent if the Reynolds number exceeds some critical value and for such flows Langhaar's expression can not be used. Sparrow[2] developed empirically an expression for calculating the developing length for laminar flow through rectangular ducts of different apsect ratios and it is $L/D = 0.02$ Re. The flow development was found to be more rapid for ducts of higher aspect ratios. Sparrow did not mention anything about the entrance condition and this expression is not valid for different entrance conditions. This expression does not give satisfactory results for turbulent flow in the ducts. Fleming, et al [3] have also developed some theoretical analysis for laminar velocity profile through ducts with arbitrary cross-sections without identifying entrance geometry. However, all these analyses were done for laminar flow and these mathematical analyses do not satisfy the results for turbulent flow in the ducts. Much work has been done for predicting the entrance length for laminar flow in ducts but much less work has been done for the developing region with turbulent flow in the ducts. Many flows at the entrance region are turbulent, specially in relation to wind tunnel entrance. It is always demanded that the velocity at the beginning of test section must be uniform. so that the wind tunnel designers always put an attention to make the entrance velocity profile development fast which depends upon the entrance geometry. This is useful for making the entrance duct short for the wind tunnel. So, the developing length in a duct is important both for laminar and for turbulent flows in many practical cases.

In the developing region of flow in pipe the velocity profile is not uniform and it is changing with axial distance. So, the momentum and kinetic energy integral equations need to be corrected as regards to the nonuniformity of the velocity profiles. In a developed turbulent flow in a pipe the correction factors for momentum and kinetic energy are assumed to be unity by many authors which is reasonable. But for laminar developed flow such factors are found to be more than unity, and their approximation to unity accumulates some error in the momentum and kinetic energy balance equations. For the developing region in a pipe both for laminar and for turbulent flows, the momentum and kinetic energy correction factos should not be counted to be unity for achieving accurate results. Coladipietro and Sridhar[4] explained the magnitude of variation of the momentum and kinetic energy correction factos in relation to a few flows both turbulent and laminar. A relation between the two factors was also developed by them with approximation. Coladipietro and Sridhar's [4] analysis for the correction factors will be examined for the developing region of pipe flow with specified entrance cone.

Hue-Shen Tien [5] has developed theoritically various stream line profiles which were useful for making converging cones for wind tunnels. The profiles were developed both for compressible and incompressible flows. One of such stream line profiles is used to make the entrance cone for a ciruclar pipe is given in ref. [6]. The present investigation is on the flow development in the developing region identifying the entrance geometry and for turbulent flow in pipes. The magnitude of the momentum and kinetic energy correction factors will also be examined. The internal surface of the pipe was polished by emery to validate the assumption that the surface is smooth.

EQUATIONS

The boundary layer velocity profile in the developing region was assumed to be,

$$\frac{u}{u_c} = \left[\frac{r_0 - r}{\delta}\right]^{1/n} \qquad (1)$$

where δ is the boundary layer thickness. The displacement thickness and momentum thickness are calculated by using the following equations.

$$\delta^*/\delta = \int_0^1 [1 - u/u_0]\, d(y/\delta) \qquad (2)$$

Where $y = r_o - r$ The non-dimensonal pressure drop is expressed in the form $\Delta P/[\, 1/2\, \rho V_a^{\,2}]$ where V_a is average velocity across the cross section of the pipe.

The momentum and kinetic energy correction factors are calculated by equations,

$$\beta = \frac{1}{A}\int (u/V_\alpha)^2\, Va$$

$$\text{and, } \alpha = \frac{1}{A}\int (u/V_\alpha)^3\, Va$$

For the velocity profile given in equation(1), α and β becomes,

$$\beta = \frac{(n+1)^2\,(2n+1)}{2n^2\,(n+2)(2n+2)} \qquad (4)$$

$$\alpha = \frac{(n+1)^3\,(2n+1)}{4n^4\,(n+3)(2n+3)} \qquad (5)$$

Coladipietro and Sridhar [4] with approximation obtained the following relationship between α and β for any type of flow.

$$\alpha = \beta^2 + \beta - 1 \qquad (6)$$

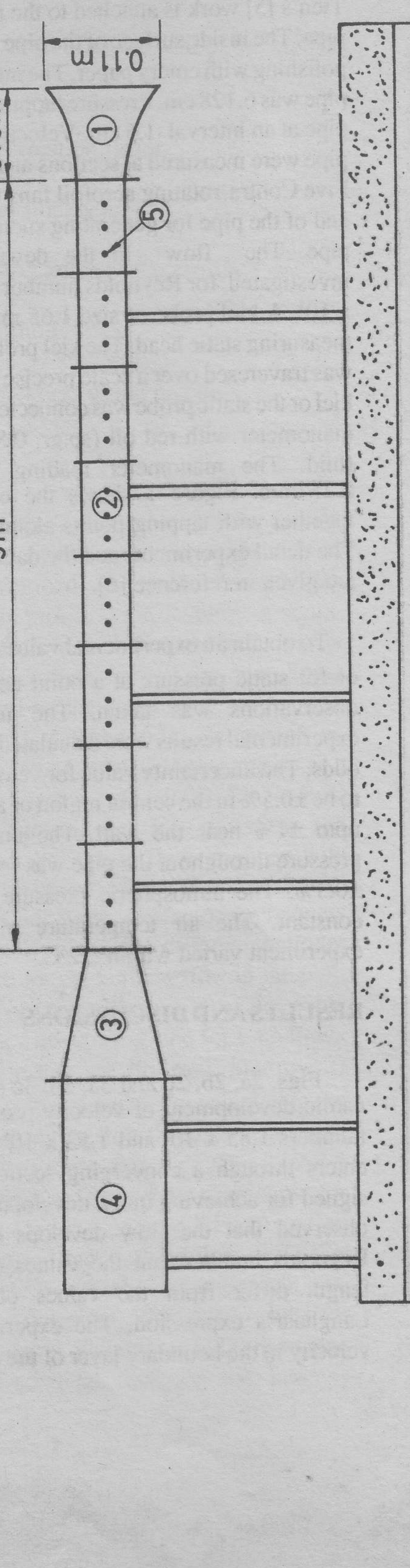
EXPERIMENTAL SET-UP AND EXPERIMENT

A converging section designed as per Hsue-Shen Tien's [5] work is attached to the inlet of a 6m long GI pipe. The inside surface of the pipe was made smooth by polishing with emery paper. The internal diameter of the pipe was 6.128 cm. Pressure tappings were set along the pipe at an interval 1.5 cm. Velocity distributions in the pipe were measured at sections at an interval of 0.15 m. Five Contra-rotating aerofoil fans were used at the exit end of the pipe for generating suction flow through the pipe. The flow in the developing region was investigated for Reynolds number 1.53×10^5 and 1.85×10^5. A kiel probe of size 1.65 mm OD was used for measuring static head. The kiel probe or the static probe was traveresed over a scale precise upto 0.254mm. The kiel or the static probe was connected to an inclined tube manometer with red oil (sp.gr. 0.827) as manometric fluid. The manometer reading was precise upto 0.254mm. Figure 1 shows the experimental set-up together with tapping points along the axial distance. The detail experiments and the data analysis procedure are given in reference [6].

To obtain an experimental value either for total head or for static pressure at a point an average of 3 to 6 observations was taken. The uncertainty of the experimental results were calculated on the basis of 20:1 odds. The uncertainty value for velocity was calculated to be ±0.5% in the central region of a section and it goes upto ±1% near the wall. The uncertainty of static pressure throughout the pipe was found to be less then ±0.3%. The atmospheric pressure was found to be constant. The air temperature in the pipe during experiment varied within ±2°C.

RESULTS AND DISCUSSIONS

Figs. 2a, 2b, 2c and 3a, 3b, 3c show the hydrodynamic development of velocity profiles for Reynolds numbers 1.85×10^5 and 1.53×10^5 respectively. Air enters through a converging section which was designed for achieving quick development of flows. It is observed that the flow develops earlier for higher Reynolds number, but the values of the developing length differ from the values obtained by using Langhaar's expression. The experimental values of velocity in the boundary layer of the developing region



1. CONVERGING NOZZLE
2. PIPE
3. DIVERGING DUCT
4. FANS
5. PRESSURE TAPS

FIG. 1 FRONT VIEW OF THE EXPERIMENTAL SET UP were fitted to the empirical equation (1) and the best value of the exponent n was calculated. The values of n for the boundary layer velocity profiles vary with the axial distance as shown in figure 4 for Re = 1.85 x 10^5 and 1.53 x 10^5. The solid lines in figs. 2a, 2b, 2c and 3a, 3b, 3c represent the best fit lines and the RMS deviation for any curve did not exceed 0.00235. The boundary layer thickness δ for $u/u_c = 0.99$ was calculated by plotting velocity in the plane u/u_c vs. r/r_o. The values of n given in figure 4 show a value about 11.0 at entrance and gradually drops to a constant value 7.6 for Reynolds number 1.85 x 10^5 at x/D = 26.5. For Reynolds number 1.53 x 10^5 the constant value of n is achieved to be 5.7 at x /D = 36. This gives an indication that the flow becomes developed at x/D = 26.5 for Re = 1.85 x 10^5 and at x/D = 36 for Re = 1.53 x 10^5. These values of developing length do not agree with the Langhaar's [1] formula for flow through circular pipes with sharp entrance. Of course, such disagreement is expected as the Langhaar's formula is applicable for laminar developing flow.

The displacement thicknesses and momentum thicknesses calculated by using equations (2) and (3) are plotted in figures 5 and 6 for Reynolds numbers 1.53X10^5 and 1.85 x 10^5 respectively. Figures 5 and 6 show that both displacement thickness and momentum thickness increase with Reynolds number in the developing region and achieve a constant value in the developed region. The shape factors also vary with the axial distance and it was found to lie between the values 1.10 and 1.16 for both Reynolds number studied here. For these values of shape factors, and the values of displacement thicknesses and momentum thicknesses shown in figures 5 and 6, the boundary layer may be assumed to be turbulent throughout the entire length of the developing region. But the velocity profile changes with axial distance in a different way for varying Reynolds numbers. Due to the change of the velocity profile the kinetic energy correction factor, α and the momentum correction factor, β should vary with the axial distance in the developing region. These factors should also depend on Reynolds numbers. The factors α and β were calculated by using equations (4) and (5) and the values are plotted in figure 7 as a function of axial distance for two Reynolds numbers. The factors are small in the entrance section and increase with the axial distance. Both α and β become maximum and constant at some axial distance which is the developed region of the flow. Figure 7 also shows that the values of the factors α and β are high if the Reynolds number become low maintaining the flow turbulent. The energy correction factor becomes as high as 1.1 for Reynolds number 1.53 x 10^5 and 1.074 for Reynolds number 1.85 x 10^5 approximately. Figure 7 also identifies the length of the developing region corresponding to the maximum constant values of α and β and it was found to be x/D=26.5 for Re= 1.85 x 10^5 and x/D = 36 for Re = 1.53 x 10^5 approximately. Figure 7 indicates that the assumption of constant values for α and β in the developing region leads to an error to the energy and momentum balance equations.

As the factors α and β are the functions of Reynolds numbers and axial distances, a simple curve fitting principle is applied to establish the relationships as shown in figure 8. The RMS error for this fitting did not exceed ±1.21 x 10^{-5} for any Reynolds number examined here. So, the empirical relationships for α and β shown in figure 8 are quite satisfactory and useful for the developing region of a pipe flow. Coladipietro and Sridhar's [4] relation between α and β is shown in figure 9 together with a comparison with the experimental values. Though the theoritical relation was developed with some approximations and assumptions, its trend agrees with the present experimental results.

SUMMARY AND CONCLUSION

1. Flow development in a pipe with specified entrance condition for different Reynolds number was studied. Developing length is shorter for higher Reynolds number.

2. The momentum and energy correction factors increase in the developing region attain maximum constant values at distance where flow becomes fully developed. The factors, α and β decrease with the increase of Reynolds number.

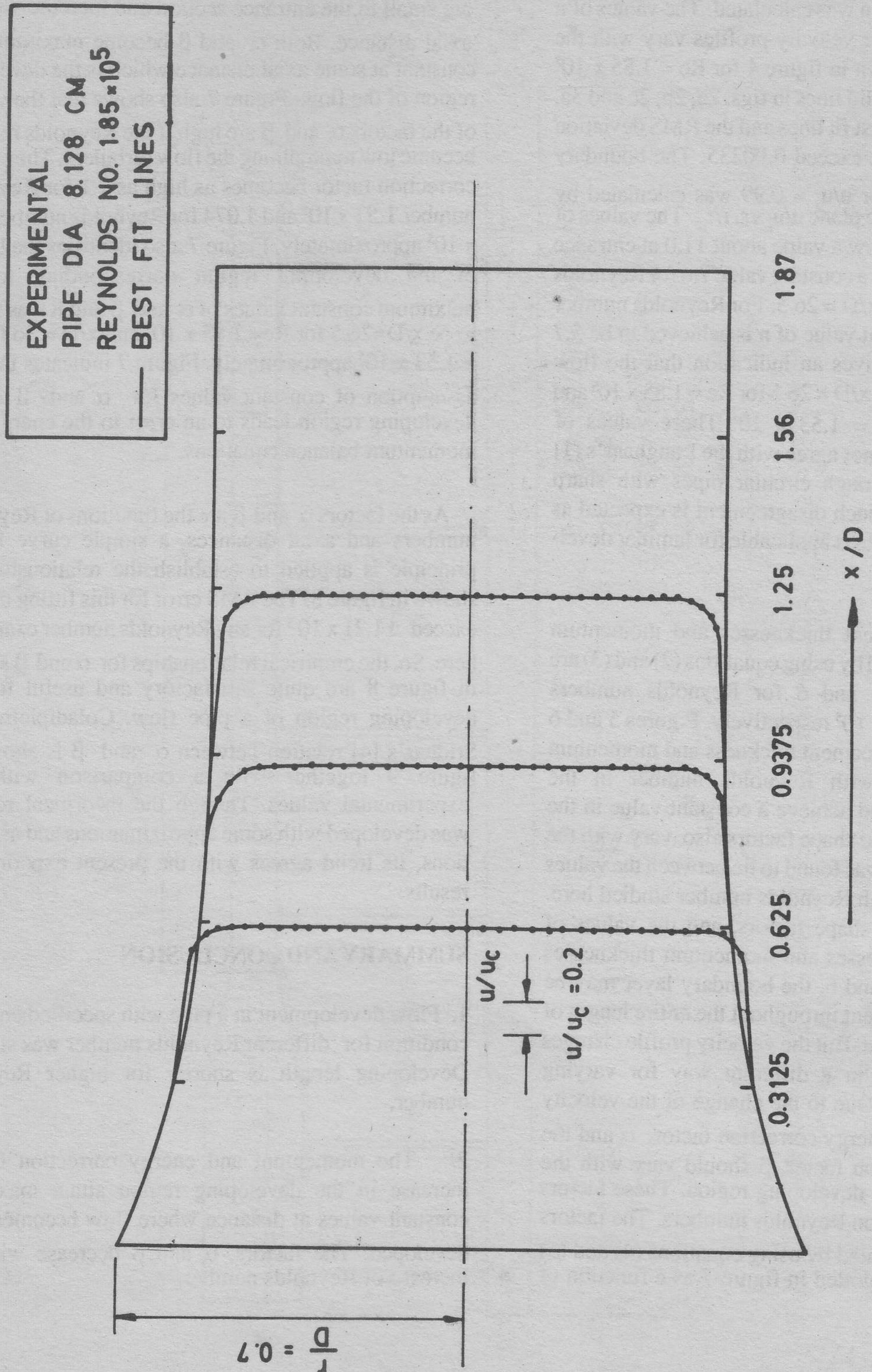


FIG. 2 (a) VELOCITY PROFILE IN THE NOZZLE

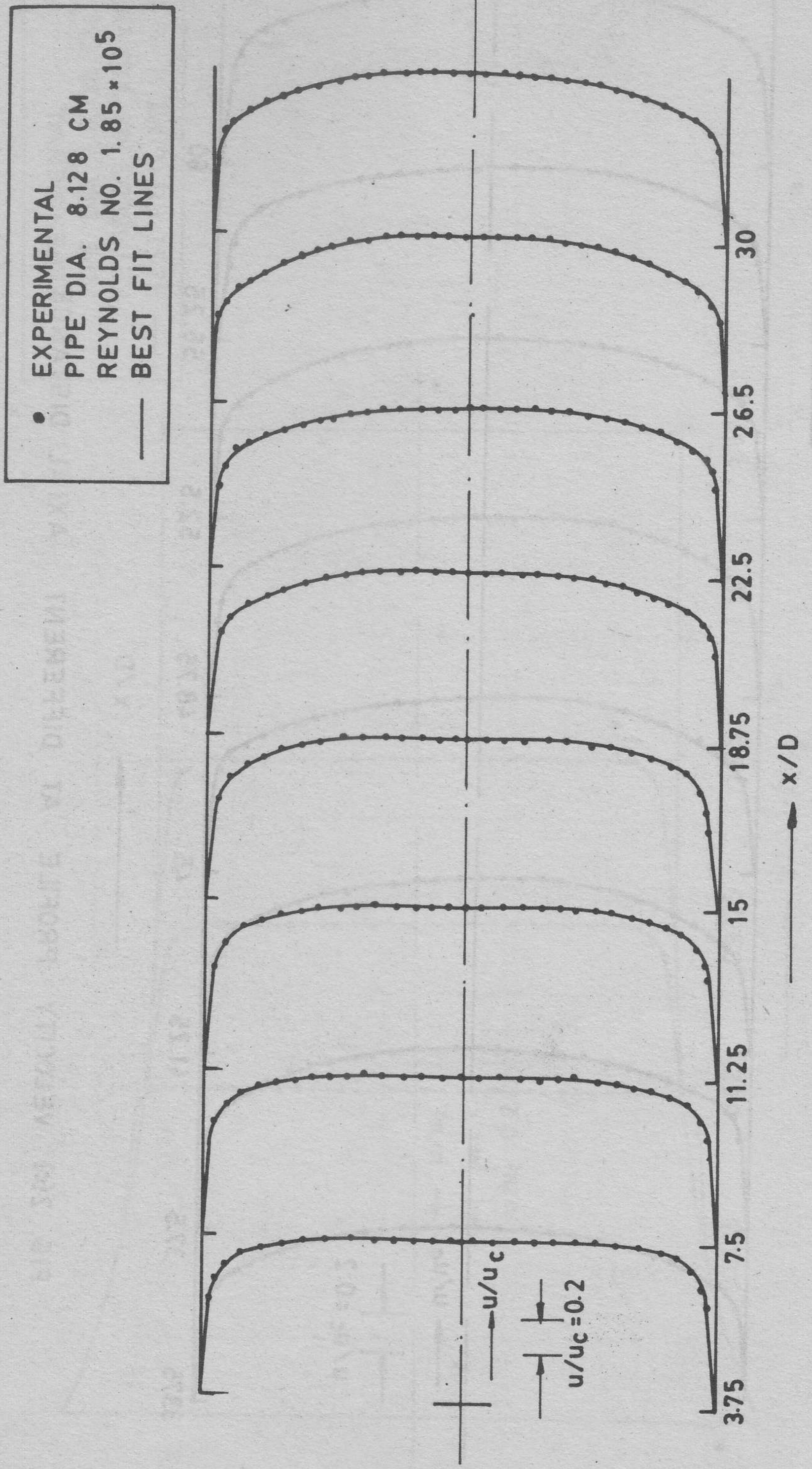


FIG. 2 (b) VELOCITY PROFILE AT DIFFERENT AXIAL DISTANCE

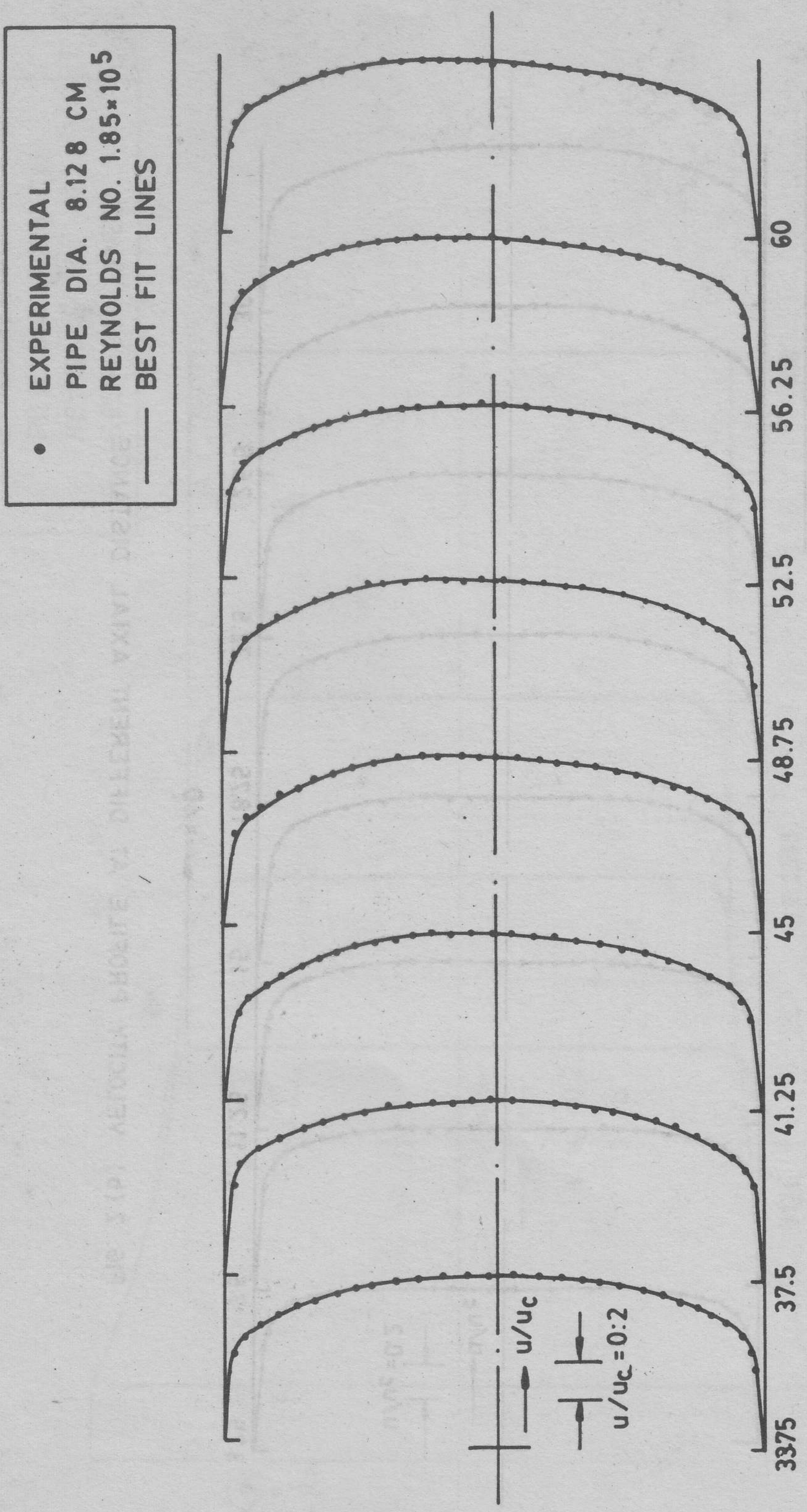


FIG. 2(c) VELOCITY PROFILE AT DIFFERENT AXIAL DISTANCE

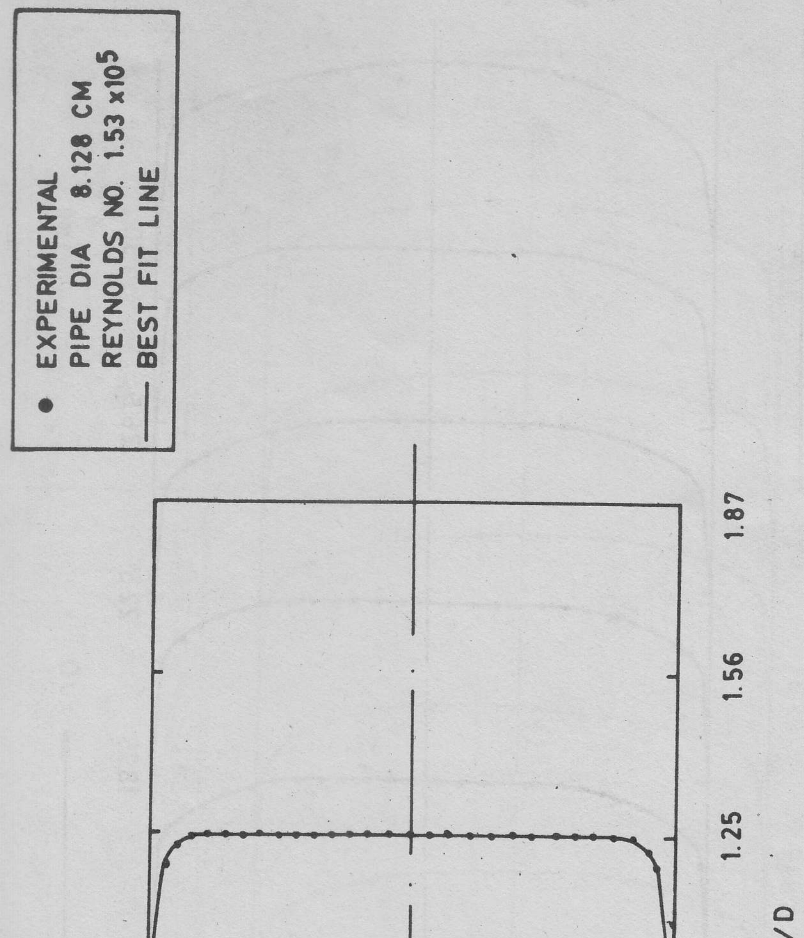


FIG. 3a VELOCITY PROFILE IN THE NOZZLE

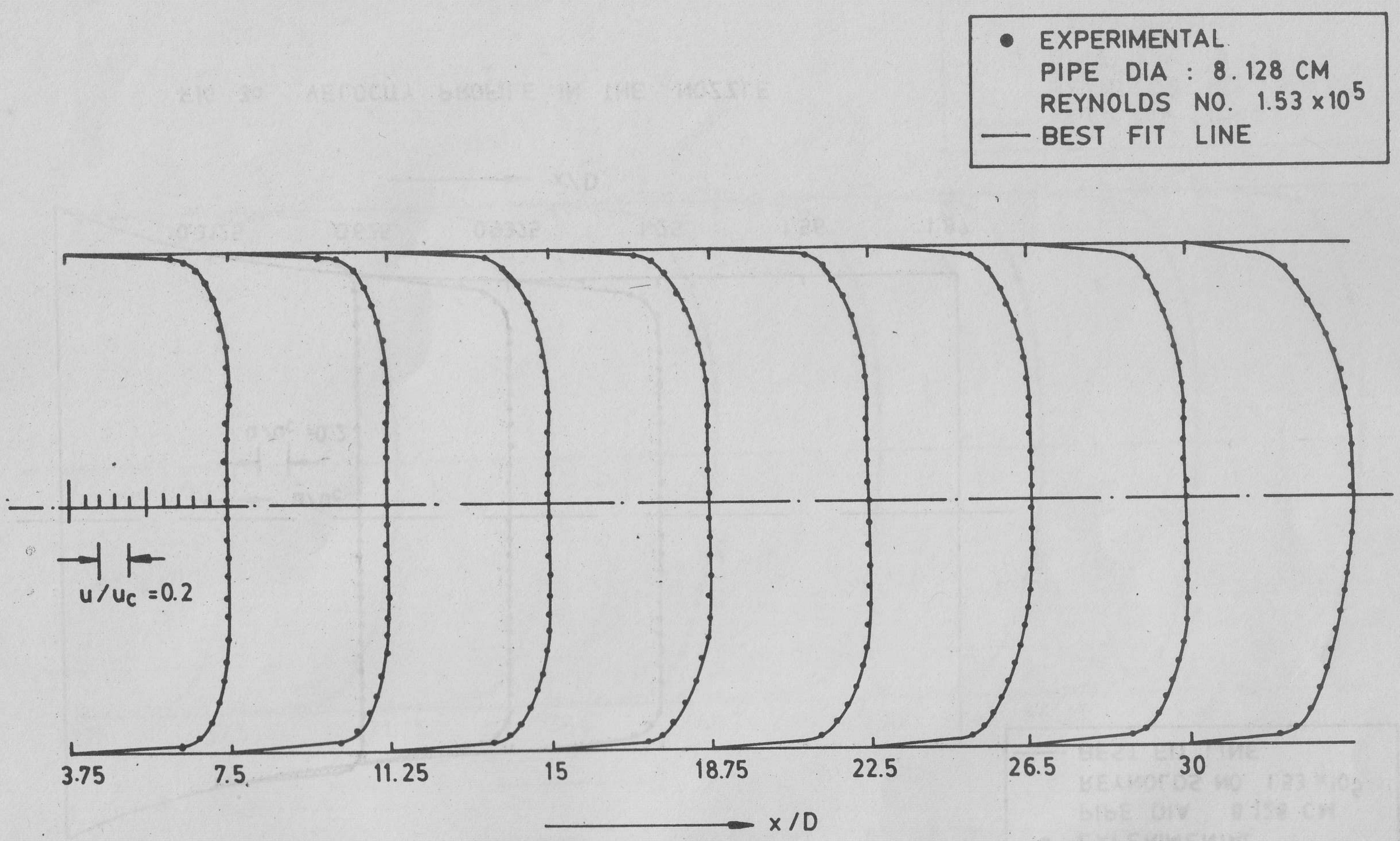


FIG. 3b VELOCITY PROFILE AT DIFFERENT AXIAL DISTANCE

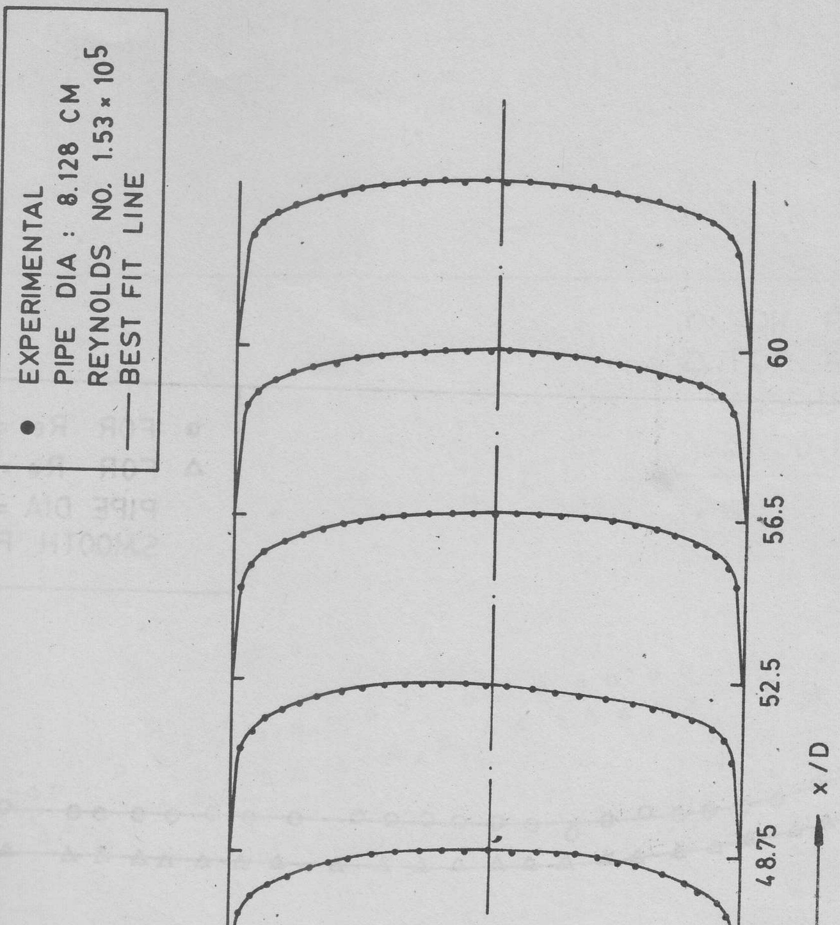


FIG 3c VELOCITY PROFILE AT DIFFERENT AXIAL DISTANCE

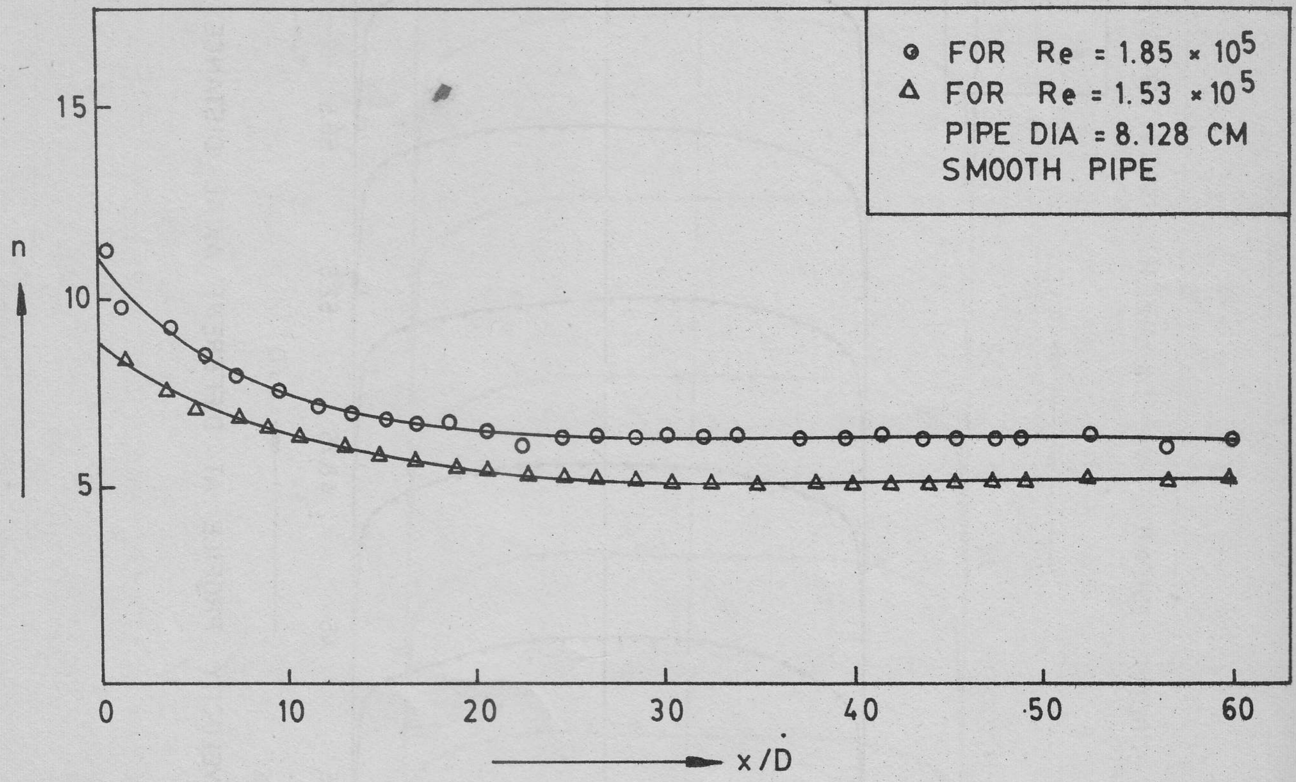


FIG. 4 VELOCITY PROFILE EXPONENT n VS AXIAL DISTANCE

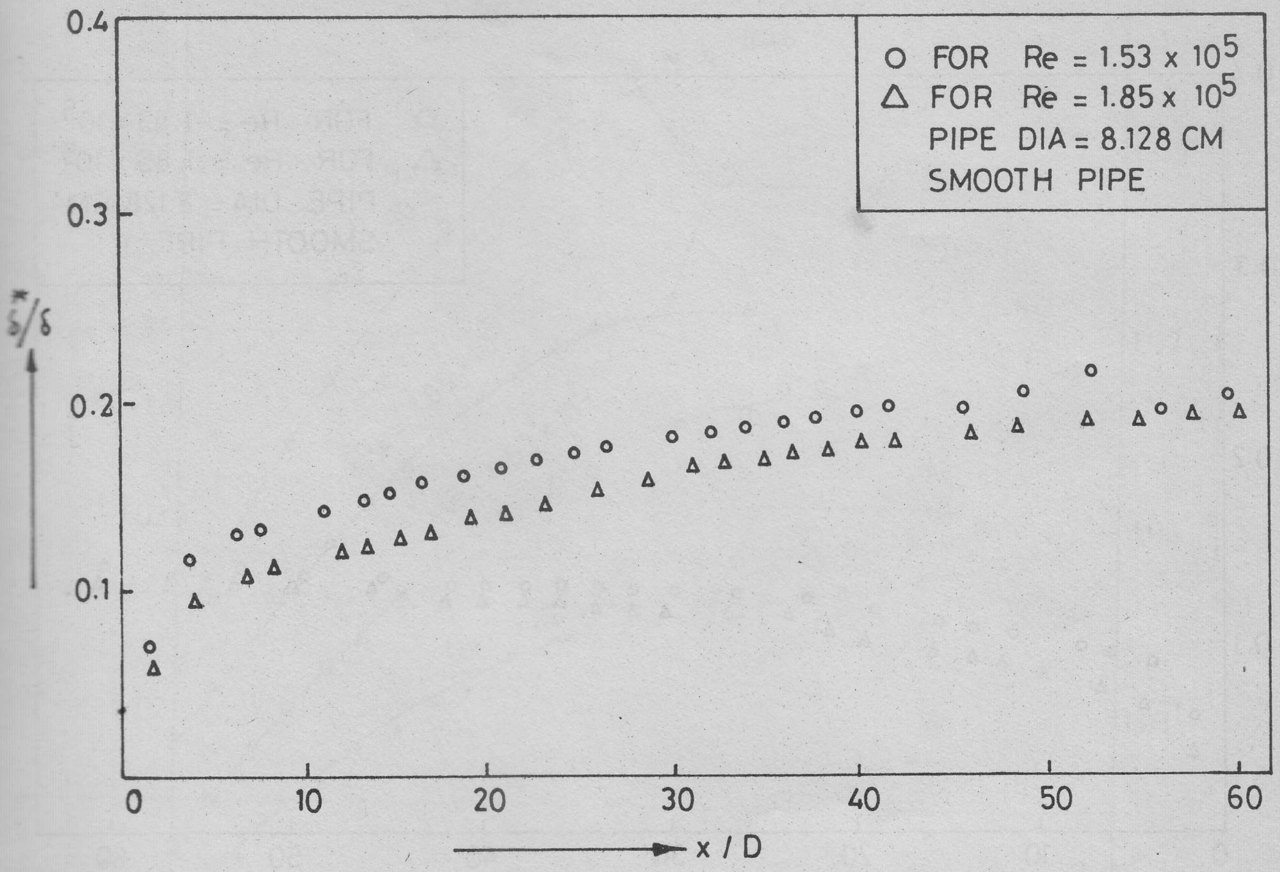


FIG. 5 DISPLACEMENT THICKNESS VS AXIAL DISTANCE IN THE CIRCULAR PIPE

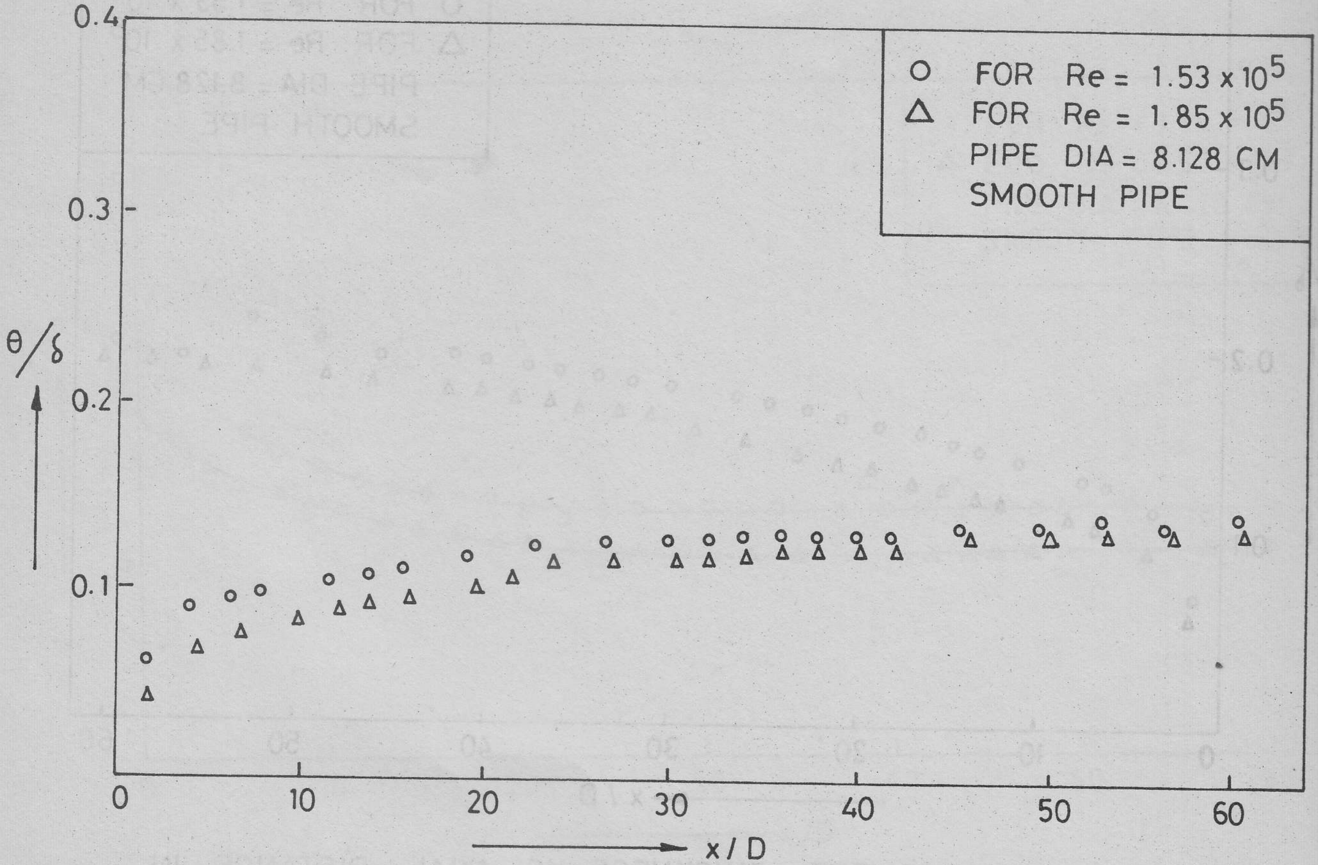


FIG. 6 . MOMENTUM THICKNESS VS AXIAL DISTANCE

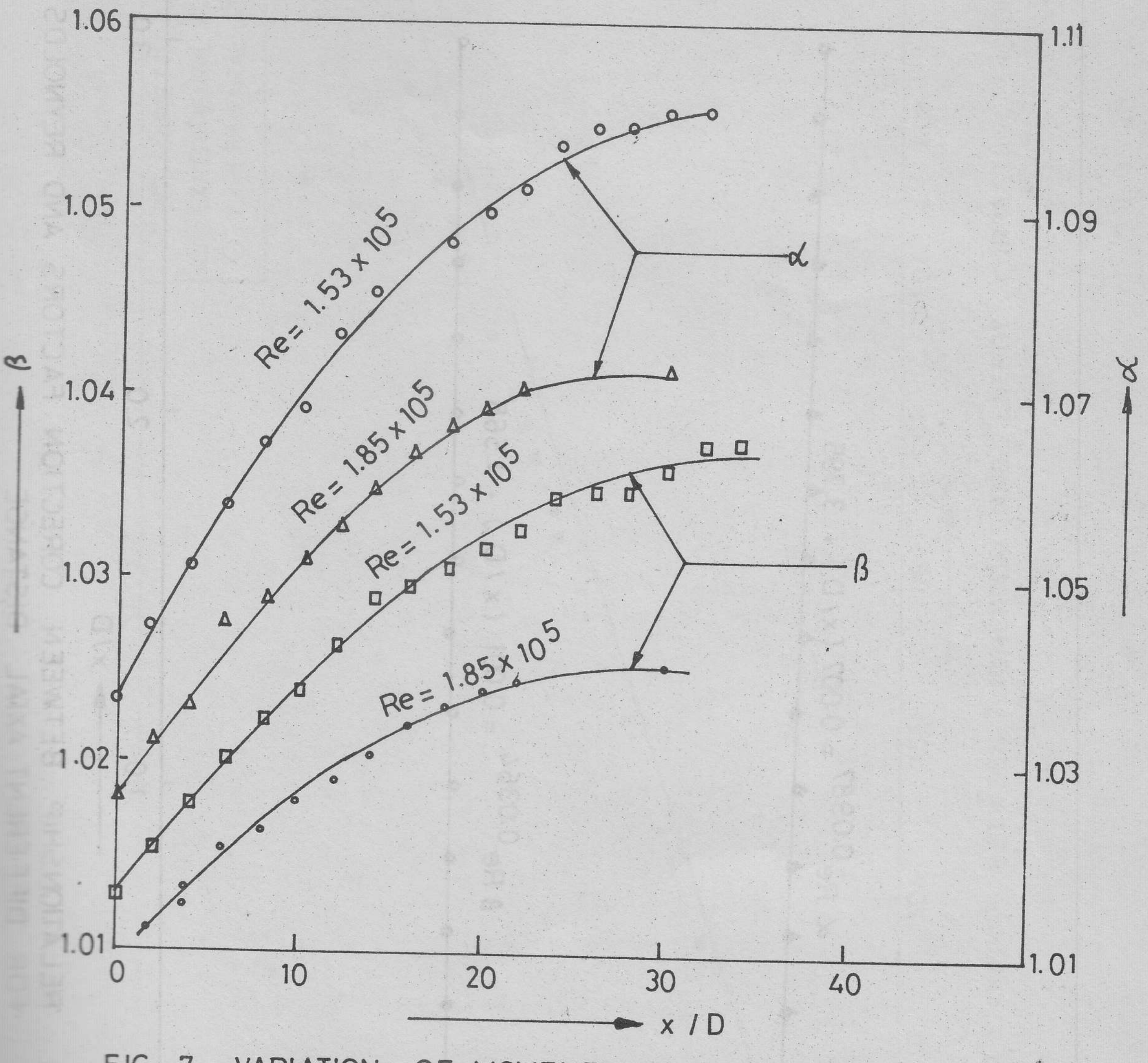


FIG. 7 VARIATION OF MOMENTUM AND ENERGY CORRECTION FACTORS WITH AXIAL DISTANCE FOR DIFFERENT REYNOLDS NUMBERS

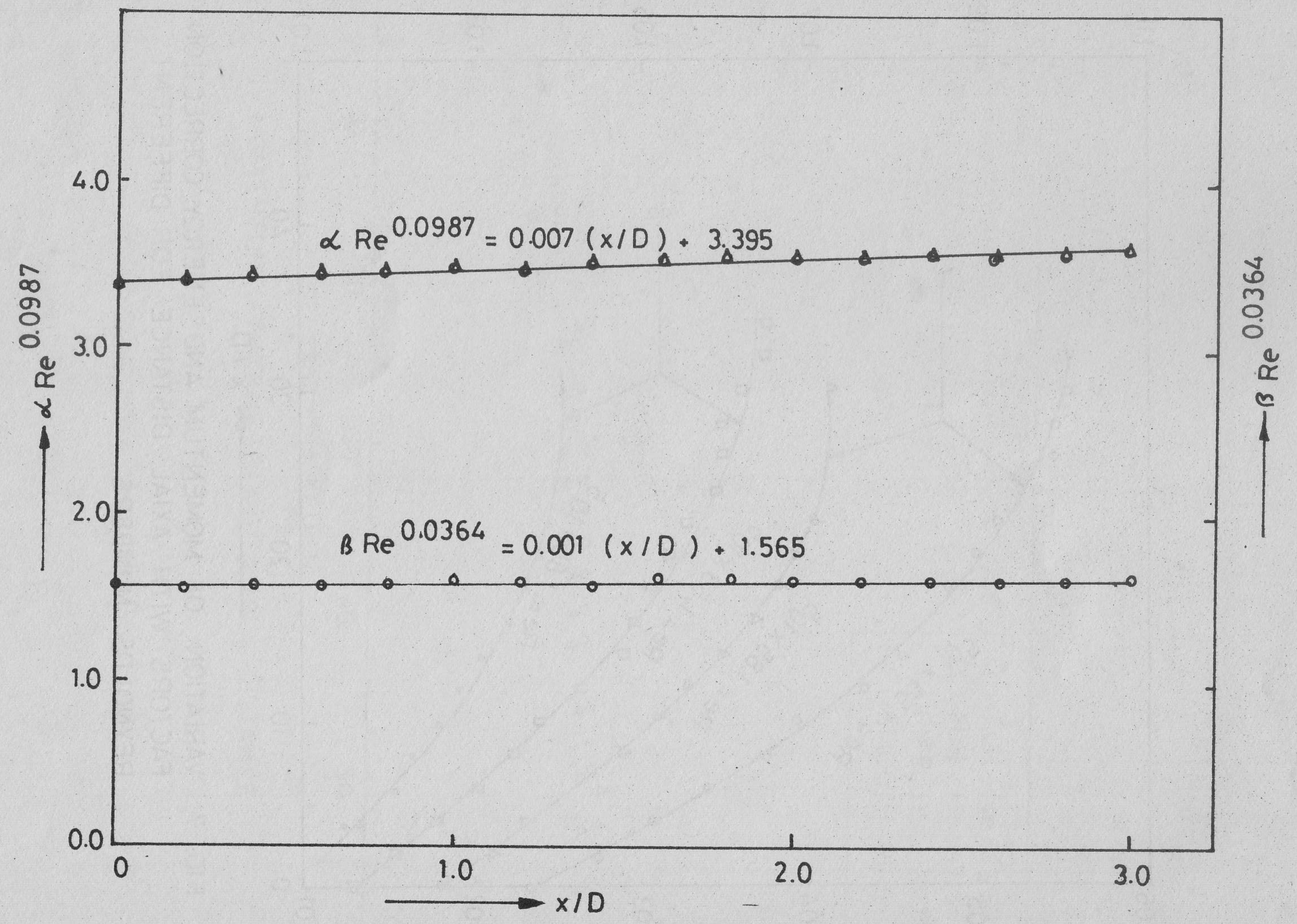


FIG. 8 RELATIONSHIP BETWEEN CORECTION FACTORS AND REYNOLDS NUMBER FOR DIFFERENT AXIAL DISTANCE

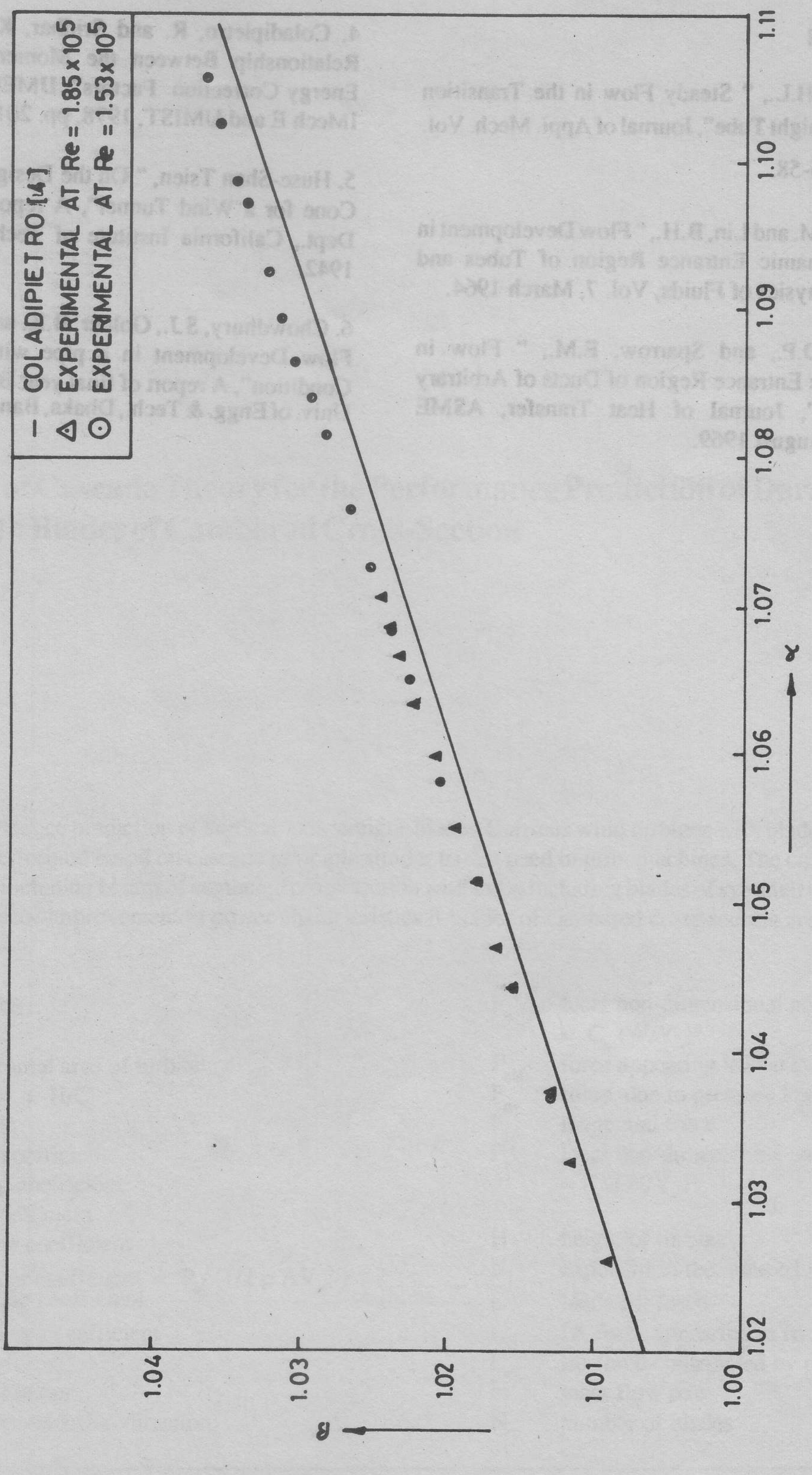


FIG. 9 RELATIONSHIP BETWEEN MOMENTUM AND ENERGY CORRECTION FACTORS

REFERENCES

1. Langhaar, H.L., " Steady Flow in the Transition Length of a Straight Tube", Journal of Appi. Mech. Vol. 9, 1942, pp. 55-58.

2. Sparrow, E.M. and Lin, B.H., " Flow Development in the Hydro-dynamic Entrance Region of Tubes and Ducts", The physics of Fluids, Vol. 7, March 1964.

3. Fleming, D.P., and Sparrow, E.M., " Flow in Hydrodynamic Entrance Region of Ducts of Arbitrary Cross-Section", Journal of Heat Transfer, ASME Transaction, August 1969.

4. Coladipietro, R. and Sridhar, K., " Approximate Relationship Between the Momentum and Kinetic Energy Correction Factors", IJMEEE, Vol. 6, No. 4, IMech E and UMIST, 1978, pp. 201 - 203.

5. Huse-Shen Tsien, " On the Design of a Contraction Cone for a Wind Tunnel", A report of Mech. Engg. Dept., California Institute of Technology, Sept. 21, 1942.

6. Chowdhury, S.J., Goldar, B.K. and Debnath, J.C., " Flow Development in a pipe with Specified Entry Condition", A report of final year B. Sc. Engg. Project, Univ. of Engg. & Tech., Dhaka, Bangladesh, May 1981.