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Performance Prediction of a Darrieus Turbine with Cascade Theory Including Blade Pitching

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ABSTRACT

A theoretical investigation of the performance prediction of a straight-Bladed Darrieus wind turbine with blade pitching is presented. Blade pitching is incorporated into the cascade theory. For any solidity of turbine, blade pitching and tip speed ratio within the usable values, performance analysis of a Darrieus turbine may be done without convergence problem associated with the conventional methods.

NOMENCLATURE

A	projected frontal area of turbine
AR	aspect ratio = H/C
C	blade chord
C_d	drag coefficient
C_p	overall power coefficient
C_Q	overal torque coefficient
D	blade drag force
F_n	normal force in radial direction
F_t	tangential force
H	height of turbine
L	blade lift force
$\dot{m}$	mass flow rate
N	number of blades
P	static pressure
Q	overall torque
R	radius of turbine
t	blade spacing = $(2\pi R/N)$
R_{e}	turbine speed Reynolds number = $R\omega C/\upsilon$
$R_{e\infty}$	wind speed Reynolds number = $V_\infty C/\upsilon$
V_a	induced velocity
V_e	wake velocity
V_Γ	velocity contributed by circulation
V_∞	wind velocity
W	relative flow velocity
W_o	relative flow velocity appearing in rectilinear flow
α	angle of attack
α_o	angle of attack appearing in rectilinear flow
γ_p	blade pitch angle
Γ	circulation per unit length
θ	azimuth angle
λ	tip speed ratio
υ	kinematic viscosity
ρ	fluid density
σ	solidity = NC/R
ω	angular velocity of turbine in rad/sec.

Subscripts

d	downstream side
u	upstream side
1	cascade inlet
2	cascade outlet

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INTRODUCTION

In order to determine the performance characteristics of a straight-bladed Darrieus wind turbine, blade pitching is added to the cascade theory presented by Hirsch and Mandal [1]. The conventional methods of simple multiple streamtube theory as suggested by Strickland [2] and multiple streamtube model with flow curvature effect including blade pitching [3] cannot predict the performance at very high solidity and high tip speed ratio and it cannot encounter any value of blade pitching because of the convergence problem indicating the limitation of the theory. Moreover, if the vortex theory [4] is applied it cannot also predict the performance of a Darrieus turbine reasonably rather it often creates convergence problem and takes very high computation time. However, the cascade theory including the blade pitching may determine performance without making any convergence problem even for a high solidity turbine, at high tip speed ratio and for any value of blade pitching within the usable range.

Aspect ratio effect is taken into account in the calculation [5]. Two dimensional lift-drag characteristics are considered in the calculation in accordance with the references [6], [7], [8] and [9]. The effect of zero lift-drag coefficient as a function of chord-radius ratio is also encountered in the calculation [10]. No correlation of calculated and experimental results are made because of the nonavailability of the experimental results with the blade pitching. However, cascade theory always show good correlation with the available experimental results without blade pitching.

AERODYNAMIC THEORY

The cascade theory presented in the reference [1] is applied to determine the performance characteristics of a straight-bladed Darrieus wind turbine. The turbine overall power coefficient is obtained from,

$$C_P = C_Q \cdot \lambda \qquad (1)$$

where C_Q is the turbine overall torque coefficient and λ is the tip speed ratio. The turbine torque coefficient may be expressed as,

$$C_Q = \frac{Q}{\frac{1}{2}\rho A V_\infty^2 R} \qquad (2)$$

Tip speed ratio may be written as,

$$\lambda = \frac{R\omega}{V_\infty} \qquad (3)$$

The expression of the overall torque coefficient may be determined in the following way. Figure 1 shows the velocities and forces on blade airfoil with pitching in cascade configuration. Along the bounding streamlines (figure 1), the pressure forces are cancelled; viscous forces can be neglected outside of the boundary layers. There exists only the momentum flux through the straight lines parallel to the cascade plane. So the force in the tangential direction due to the rate of change of momentum is obtained as,

$$F_t = \dot{m}\,(W_2 \cos\alpha_2 - W_1 \cos\alpha_1) \qquad (4)$$

Applying continuity equation, the mass flow rate $\dot{m}$ can be found as,

$$\dot{m} = \rho t W_1 \sin\alpha_1 H = \rho t W_2 \sin\alpha_2 H = \rho W_x H \qquad (5)$$

From the equations (4) and (5) the tangential force F_t becomes,

$$F_t = \rho t (W_2^2 \sin\alpha_2 \cos\alpha_2 - W_1^2 \sin\alpha_1 \cos\alpha_1) H \qquad (6)$$

Tangential force represented by the equation (6) is for any azimuthal position, so writing $t = 2\pi R/N$ in the equation (6), one may obtain,

$$F_t(\theta) = \rho \frac{2\pi R}{N} (W_2^2 \sin\alpha_2 \cos\alpha_2 - W_1^2 \sin\alpha_1 \cos\alpha_1) H \qquad (7)$$

Average tangential force on one blade for blade length of H, may be written as,

$$F_{ta} = \frac{1}{2\pi} \int_0^{2\pi} F_t(\theta)\, d\theta \qquad (8)$$

The blade torque for number of blades N on blade length of H is obtained as,

$$Q = N F_{ta} . R \tag{9}$$

Now from the equations (2), (7), (8) and (9), one may derive the expression of overall torque coefficient as,

$$C_Q = \int_0^{2\pi} \left(\frac{W_2^2}{V_\infty^2} \sin\alpha_2 \cos\alpha_2 - \frac{W_1^2}{V_\infty^2} \sin\alpha_1 \cos\alpha_1 \right) d\theta \tag{10}$$

where W_1, W_2 and α_1, α_2 are found as in accordance with the reference [1],

$$\frac{W_1^2}{V_\infty^2} = \frac{W_x^2}{V_\infty^2} + \frac{(W_y - V_\Gamma)^2}{V_\infty^2} \tag{11}$$

$$\frac{W_2^2}{V_\infty^2} = \frac{W_x^2}{V_\infty^2} + \frac{(W_y + V_\Gamma)^2}{V_\infty^2} \tag{12}$$

$$\alpha_1 = \tan^{-1} \left[\frac{W_x / V_\infty}{(W_y - V_\Gamma) / V_\infty} \right] \tag{13}$$

$$\alpha_2 = \tan^{-1} \left[\frac{W_x / V_\infty}{(W_y + V_\Gamma) / V_\infty} \right] \tag{14}$$

The expression of the wake velocity ratio may be obtained as [1],

$$\frac{V_e}{V_\infty} = \sqrt{1 - \left(\frac{W_2^2}{V_\infty^2} - \frac{W_1^2}{V_\infty^2} \right) - \frac{1}{2\pi} \frac{NC}{R} \frac{C_d}{\sin\alpha_o} \frac{W_o^2}{V_\infty^2}} \tag{15}$$

Induced velocity ratio is written in the form,

$$\frac{V}{V_\infty} = \left(\frac{V_e}{V_\infty} \right)^{k_i} \tag{16}$$

where the exponent k_i may be determined from,

$$k_i = (.425 + .332\sigma) \tag{17}$$

The velocity ratios for both the upstream and downstream sides may be calculated from the similar equations given above. However, while calculating the induced velocity ratio downstream side, the wake velocity of upstream side is assumed as the free stream velocity for the downstream side.

To encounter the effect of blade pitching the angle of attack value is needed to be altered from that without blade pitching. In the analysis pitching is considered to be positive for the blade airfoil nose rotating in the outward direction from the blade flight path. As a result for upstream side the angle of attack becomes,

$$\alpha_u = \alpha_{ou} - \gamma_{pu} \tag{18}$$

and that for the downstream side,

$$\alpha_d = \alpha_{od} + \gamma_{pd} \tag{19}$$

where γ_{pu} and γ_{pd} are the pitch angles in the upstream and the downstream sides respectively. α_{au} is expressed as [1],

$$\alpha_{ou} = \tan^{-1} \left[\frac{\sin\theta}{\frac{R\omega}{V_\infty} \Big/ \frac{V_{au}}{V_\infty} + \cos\theta} \right] \tag{20}$$

α_{od} may be obtained from the equation (20) replacing the subscript u by d.

The lift and drag characteristics are taken corresponding to the α_u (upstream) and α_d (downstream). The parameters shown in the figure 1 have not been subscripted to make them generalized. Subscripts u and d are used with the parameters for the upstream and the downstream sides respectively.

Iteration process is applied in order to calculate the velocity ratios used in the equation (10). Induced velocity ratios for upstream and downstream sides are calculated separately. For the known values of tip speed ratio, solidity and azimuth angle, the starting value of induced velocity ratio is either chosen as unity or as that calculated for the prior station. Now the new value of the induced velocity ratio is determined by using the equation (16). This process is continued until the induced velocity ratio is obtained with desired accuracy. The power coefficient (equation 1) is calculated by numerical integration method with the help of Sympson's rule.

RESULTS AND DISCUSSIONS

Performance characteristics of straight-bladed Darrieus wind turbines with blade pitching have been presented in the figure 2 to 5 of which the figures 2 and 3 show respectively the comparisons of the calculated values of overall power and torque coefficients by both cascade theory and multiple streamtube theory with flow curvature effect [3] at various fixed pitching (positive). The results in these figures are shown employing pitching with the West Virginia University outdoor test model [11]. It is observed from the figures 2 and 3 that differences between the calculated values by cascade theory and those by multiple streamtube theory with flow curvature effect are not appreciable. It is also observed that with the application of fixed pitching, the rotor power always decreases. The higher is the pitching, the lower is the power coefficient. The figure 3 shows that with the variation of blade fixed pitching, there is very negligible variation of the torque at very low tip speed ratio. It indicates that incorporating fixed pitching, starting torque of a turbine cannot be improved. It has also been investigated theoretically that employing negative fixed pitching, there occurs negligible change in starting torque.

Incorporating fixed blade pitching (positive), the angle of attack decreases in the upstream and increases in the downstream sides. So the blade airfoil lift coefficient drops in the upstream side and rises in the downstream side which are the outcome of lower tangential force coefficients in the upstream and higher tangential force coefficients in the downstream sides in general. But the increased angles of attack in the downstream side sometimes go beyond stalling angle which again are the causes of reduced tangential force coefficients. However, the net effect always reduces the power coefficients.

Figures 4 and 5 show that the amplitude of angular pitch variation (sinusoidal) has significant effects in the performace curves. The figure 4 reveals that as the amplitude increases power rises with increase of amplitude in lower range while with further increase of amplitude, power begin to fall considerably. The figure 5 shows that as the amplitude increases there occurs remarkable increase of the torque at very low tip speed ratio indicating the increase of starting torque as well. In these figures performance curves with 12 degree amplitude is for the Pinson cycloturbine of U.S.A. [12]. Other curves are plotted with various amplitudes of sinusoidal pitch variations for comparative study. The calculation is done by cascade theory only, since with higher solidity, with higher blade pitching and at higher tip speed ratio mementum theory cannot give reasonable Convergence. From the figure 2 it is seen that with the

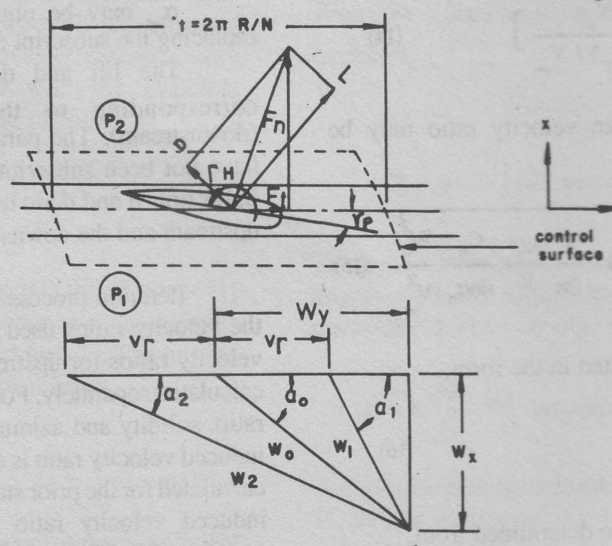

Figure 1 : Velocities and forces on blade airfoil with pitching in cascade configuration.

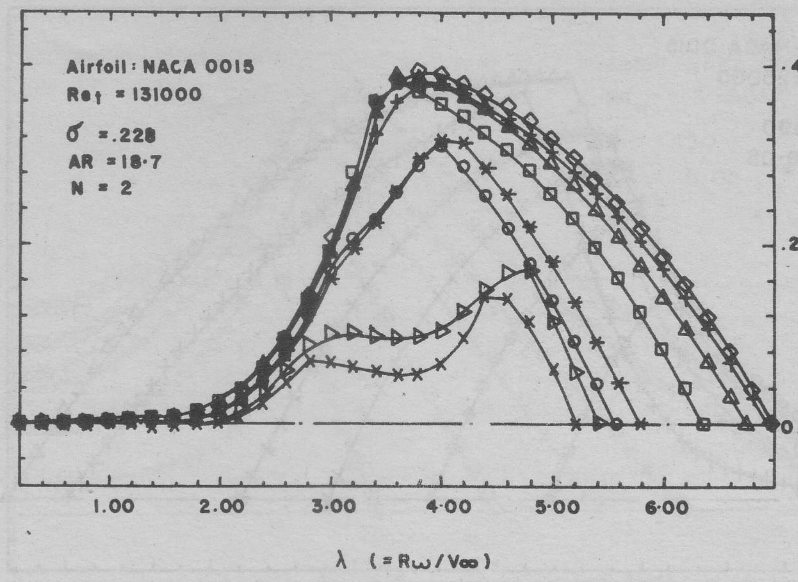

Figure 2 : Variations of overall power coefficients with tip speed ratios at different fixed blade pitchings.

calculated :	(cascade)				(multiple streamtube with curvature, Ref. [3])			
symbol :	+	Δ	*	x	◇	□	o	▷
pitch(deg.):	0	2	5	7	0	2	5	7

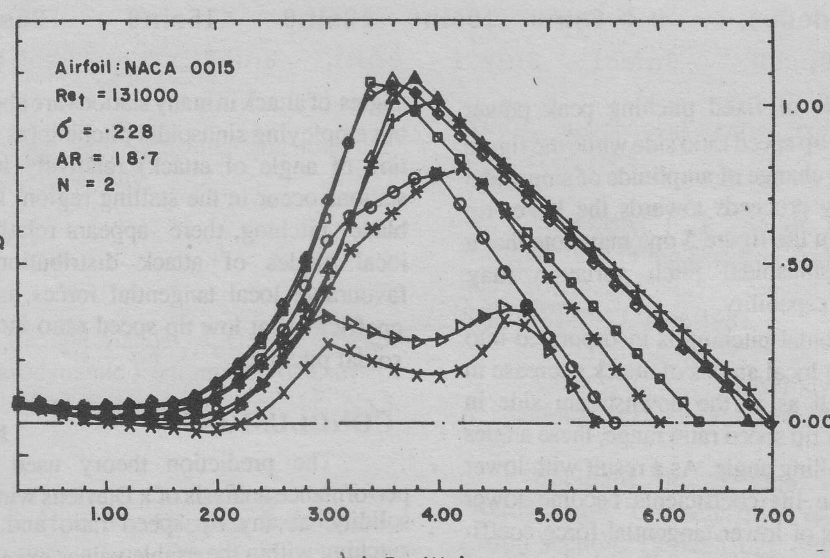

Figure 3 : Variations of overall torque coefficients with tip speed ratios at different fixed blade pitchings.

calculated :	(cascade)				(multiple streamtube with curvature, Ref. [3])			
symbol :	+	Δ	*	x	◇	□	o	▷
pitch(deg.):	0	2	5	7	0	2	5	7

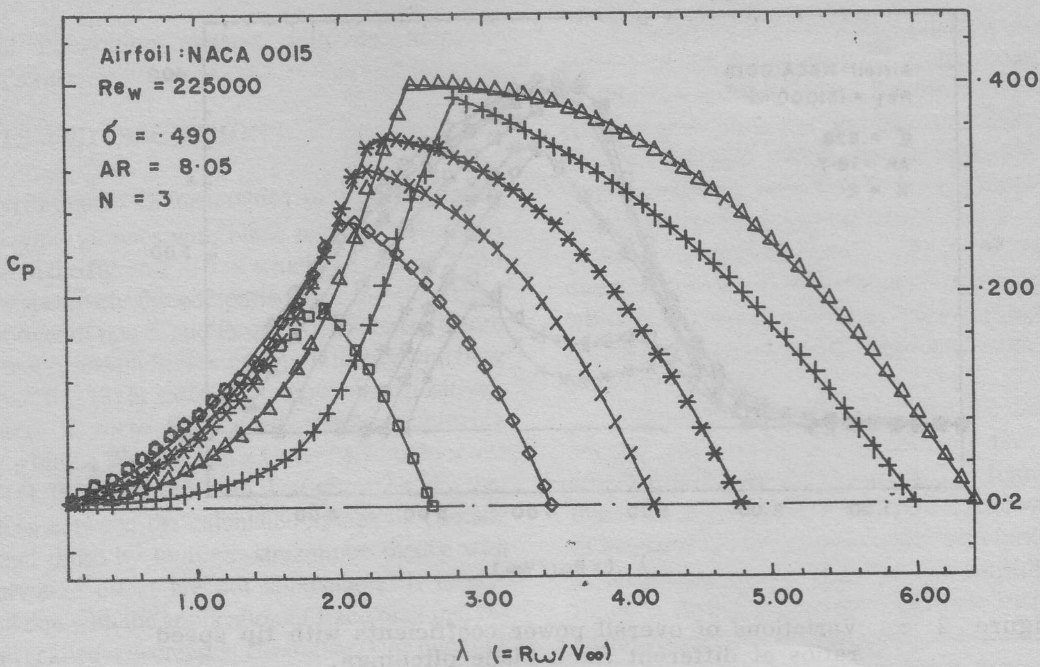

Figure 4 : Variations of overall power coefficients with tip speed ratios at different amplitudes of sinusoidal pitch variation.

symbol :	+	Δ	*	x	◇	□
γ_p (deg.) :	0	5sinθ	10sinθ	12sinθ	15sinθ	20sinθ

change of magnitude of fixed pitching peak power remains in the higher tip speed ratio side while the figure 4 shows that with the change of amplitude of sinusoidal pitching, it gradually proceeds towards the lower tip speed ratio side. From the figure 5 one may note that a cycloturbine with sinusoidal pitch variation may develop self-starting capability.

As the sinusoidal pitching is incorporated into the turbine blade, the local angles of attack decrease in the upstream as well as in the downstream side in general. In the higher tip speed ratio range, these angles remain below the stalling angle. As a result with lower angles of attack, the lift coefficients become lower which are the results of lower tangential force coefficients. So the power coefficients fall with the rise of amplitude of sinusoidal blade pitching. But there appear the exception at very low pitching as the figure 4 shows. According to this figure at low tip speed ratio range, the power coefficient increases with the rise of amplitude of pitching. It is because, at zero pitching, angles of attack in many stations are above stalling angle but employing sinusoidal pitching (as a result of reduction of angle of attack) relatively lower number of stations occur in the stalling region. Incorporating the blade pitching, there appears relatively favourable local angles of attack distribution which makes favourable local tangential forces, as a result torque coefficients at low tip speed ratio increases with sinusoidal pitching.

CONCLUSIONS

The prediction theory used may determine performance analysis of a Darrieus wind turbine for any solidity, at any tip speed ratio and with any blade pitching within the usable values even it does not make any convergence problem.

The theoretical limit of induced velocity ratio for momentum theory is 0.5 while in this method this value may go below 0.5 depicting real picture of induced velocity.

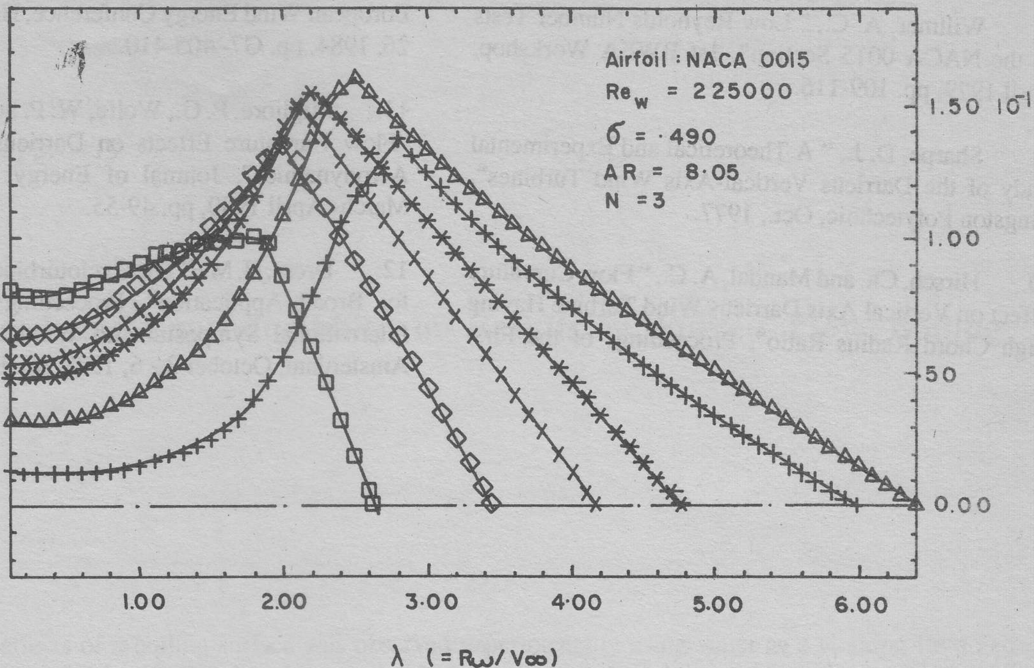

Figure 5 : Variations of overall torque coefficients with tip speed ratios at different amplitudes of sinusoidal pitch variation.

symbol :	+	Δ	*	x	◇	□
γ_p (deg.):	0	5sinθ	10sinθ	12sinθ	15sinθ	20sinθ

A cycloturbine with sinusoidal pitching may develop self-starting capability.

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