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Theoretical Structural Analysis of a Horizontal Axis Wind Turbine

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ABSTRACT

In this paper a simple method to determine the structural analysis of a two-bladed horizontal axis wind turbine is presented. The analysis considers the problems of wind shear, tower shadow, coning, tilting, yaw, centrifugal and gravity loadings. The numeri cal results obtained with the present method agree well with experimental and other numerical data.

NOMENCLATURE

C_{mp}	pitching moment coefficient, $(M_p)/(1/2\, r AV^2_a R)$
C_{mz}	moment coefficient, $(M_{yaw})/(1/2\, r AV^2_a R)$
E	modulus of elasticity
F_{x_0}	force acting along x-direction in S_o co-ordinate system
F_{x_3}	force acting along x-direction in S_3 co-ordinate system
F_{y_0}	force acting along y-direction in S_o co-ordinate system
F_{y_3}	force acting along y-direction in S_3 co-ordinate system
F_{z_0}	force acting along z-direction in S_o co-ordinate system
F_{z_3}	force acting along z-direction in S_3 co-ordinate system
g	acceleration due to gravity
i,j,k	unit vectors along x-y-, and z - direction respectively
K_θ	transformation matrix for azimuth
K_β	transformation matrix for coning
K_T	transformation matrix for tilting
M_P	pitching moment
M_{yaw}	yawing moment
m	mass of the blade
r	local blade radius
R	rotor radius
S_o	fixed reference co-ordinate system
S_1	co-ordinate system considering tilt angle
S_2	co-ordinate system relating to blade azimuth
S_3	co-ordinate system relating to blade coning
V_a	undisturbed wind velocity

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X_o	distance along x-axis in S_o reference system
Y_o	distance along y-axis in S_o reference system
Z_o	distance along z-axis in S_o reference system
α_T	tilt angle
θ	azimuth angle
β	coning angle
β_T	twist angle
ρ	air density
λ	tip speed ratio
σ	solidity
Ω	angular velocity of rotor
ϕ	angle of relative wind velocity

1. INTRODUCTION

Large wind turbine must be designed for structural efficiency and reliability together with minimum maintenance and weight is order to harness energy at a competitive rate. For the structural analysis the horizontal axis wind turbine is considered having a vertical tower with a power unit and a two-bladed into two components. One acting in the plane of the rotor and the other acting normal to the rotor plane. The former is termed as flapwise and the latter as edgewise on account of their orientation relative to that of the airfoil section. The problem of blade bending is a very complex one, for not only that the blade has several degrees of bending but for the aerodynamic loading which depends strongly on the blade shape. However, for the purpose of simplicity, the flapwise and lagwise bending of the blade are considered here sepearately.

2. FORCES AND MOMENTS (Aerodynamic)

The major forces prevailing in the operation of a horizontal axis wind turbine are the aerodynamic, gravity, centrifugal and gyroscopic forces which act on the rotor and the tower. In the present analysis, gyroscopic force has been neglected because of its smaller values in comparison to other forces for rigid tower.

Before proceeding with the analysis it is necessary to find the expression for the forces and moments in different coordinate systems. Four coordinate systems [1] are required for the analysis. These are discussed in Appendix A.

2.1 Forces

All the forces in the local S_3 coordinate system can be expressed as

$$\bar{F}_{S_3} = \begin{vmatrix} F_{x_3} \\ F_{y_3} \\ F_{z_3} \end{vmatrix} \quad \ldots \ \ldots \ \ldots (2.1)$$

Refering to the non-rotating system (S_1) attached to the hub, the expression for forces become

$$\bar{F}_{S_1} = \left[K_\theta\right]\left[K_\beta\right]\left[F_{S_3}\right] \quad (2.2)$$

The following equation yields from equation (2.2)

$$\bar{F}_{S_1} = \begin{bmatrix} F_{x_3}\,\mathrm{Cos}\theta + F_{y_3}\,\mathrm{Sin}\theta\,\mathrm{Sin}\beta + F_{z_3}\,\mathrm{Sin}\theta\,\mathrm{Cos}\beta \\ F_{y_3}\,\mathrm{Cos}\beta - F_{z_3}\,\mathrm{Sin}\beta \\ -F_{x_3}\,\mathrm{Sin}\theta + F_{y_3}\,\mathrm{Sin}\beta\,\mathrm{Cos}\theta + F_{z_3}\,\mathrm{Cos}\theta\,\mathrm{Cos}\beta \end{bmatrix} \quad \ldots\ldots \ \ldots\ldots (2.3)$$

At the tower top, forces can be expressed as,

$$\bar{F}_{S_0} = \left[K_T\right]\left[K_\theta\right]\left[K_\beta\right] \quad \text{....... ....} \ (2.4)$$

Equation (2.4) can be expressed as

$$\bar{F}_{S_1} = \left| \begin{matrix} F_{X_3} \ Cos\theta + F_{y_3} \ Sin\theta \, Sin\beta + F_{z_3} \ Sin\theta \ Cos\beta \\ F_{x_3} \ Sin\alpha_T \, Sin\theta + F_{y_3} \ (Cos\beta \ Cos\alpha_T + Sin\alpha_T \ Sin\beta \, Cos\theta) \\ -F_{z_3} \ (Cos\alpha_T Sin\beta + Sin\alpha_T Cos\beta \ Cos\theta) \\ -F_{x_3} \ Cos\alpha_T + F_{y_3} \ (Cos\beta \, Sin\alpha_T + Cos\alpha_T Sin\beta \, Cos\theta) \\ -F_{z_3} \ (Cos\alpha_T Cos\beta Cos\theta \ - \ Sin\beta \, Sin\alpha_T) \end{matrix} \right| \quad \text{...... .....} \ (2.5)$$

2.2 Moments

At the blade supporting point the expression for moment for a differential element can be written as

$$d\bar{M}_{S_3} = d\bar{F}_{S_3} * \bar{r}_{S_3} \quad \text{...... ....} \ (2.6)$$

This equation can be expressed as

$$d\bar{M}_{S_3} = \begin{bmatrix} i_3 & j_3 & k_3 \\ dF_{x_3} & dF_{y_3} & dF_{z_3} \\ 0 & 0 & r_3 \end{bmatrix} \quad \text{..... ....} (2.7)$$

where r_3 is the distance from the blade root along Z_3 direction. The equation (2.7) can be reduced to

$$\begin{bmatrix} dM_{x_3} \\ dM_{y_3} \\ dM_{z_3} \end{bmatrix} = \begin{bmatrix} r_3 dF_{y_3} \\ -r_3 dF_{x_3} \\ 0 \end{bmatrix} \quad \text{......... ....} \ (2.8)$$

The equations for total moments in different directions for one blade can be writtenas follows :

Flapwise moment,

$$M_{x_3} = \int_0^R r_3 \ dF_{y_3} \quad \text{....... ....} \ (2.9)$$

Edgewise moment,

$$M_{y_3} = \int_0^R r_3 \, dF_{x_3} \quad \ldots\ldots \ldots\ldots \ldots\ldots \ldots\ldots \ldots\ldots \ldots \text{(2.10)}$$

At the tower top corresponding to inertial system S_o, the moment can be expressed as

$$\bar{M}_{s_0} = \bar{F}_{s_0} * \bar{r}_{s_0} \quad \ldots \ldots\ldots \ldots \ldots \ldots\ldots \text{(2.11)}$$

The following equation yields from equation (2.11)

$$\begin{bmatrix} M_{x_0} \\ M_{y_0} \\ M_{z_0} \end{bmatrix} = \begin{bmatrix} i_0 & j_0 & k_0 \\ F_{x_0} & F_{y_0} & F_{z_0} \\ X_0 & Y_0 & Z_0 \end{bmatrix} \quad \ldots\ldots \ldots \ldots \ldots\ldots \text{(2.12)}$$

where X_o, Y_o, and Z_o are the moment arms in the respective coordinate system. The equation (2.12) can be expressed as

$$\begin{bmatrix} M_{x_0} \\ M_{y_0} \\ M_{z_0} \end{bmatrix} = \begin{bmatrix} Z_0 F_{y_0} & - & Y_0 F_{z_0} \\ X_0 F_{z_0} & - & Z_0 F_{x_0} \\ Y_0 F_{x_0} & - & X_0 F_{y_0} \end{bmatrix} \quad \ldots\ldots \ldots\ldots \text{(2.13)}$$

where M_{x_o}, M_{z_o} and M_{y_o} are called the pitching, yawing and rolling moments respectively.

2.3 Gravity Force

Due to self weight of the blade a direct cyclic stress is present which alternates from being tensile to compressive. Further, blade coning, tilting and azimuth introduce an additional cyclic stress due to bending moment in the flapwise and edgewise directions. Gravity force can be written in the S_o coordinate system as

$$\bar{F}_{GS_0} = \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \quad \ldots\ldots \ldots\ldots \ldots\ldots \ldots \text{(2.14)}$$

In the S_2 coordinate system force due to gravity can be written as

$$\bar{F}_{GS_2} = \left[k_\theta\right]^T \left[k_T\right]^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix}$$

or,

$$\begin{bmatrix} F_{xG_2} \\ F_{yG_2} \\ F_{zG_2} \end{bmatrix} = \begin{bmatrix} -mg\, Sin\theta\, Sin\alpha_T \\ mg\, Sin\alpha_T \\ mg\, Cos\theta\, Cos\alpha_T \end{bmatrix} \qquad \text{...... .... (2.15)}$$

In the rotating S_3 coordinated system equation (2.15) becomes

$$\bar{F}_{GS_3} = \left[k_\beta\right]^T \left[k_\theta\right]^T \left[k_T\right]^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \qquad \text{...... .... (2.16)}$$

Equation (2.16) can be expressed as

$$\begin{vmatrix} F_{xG_3} \\ F_{yG_3} \\ F_{zG_3} \end{vmatrix} = \begin{vmatrix} -mg\, Sin\theta\, Sin\alpha_T \\ mg\, Cos\beta\, Sin\alpha_T + mg\, Cos\theta\, Cos\alpha_T\, Sin\beta \\ mg\, Sin\beta\, Sin\alpha_T + mg\, Cos\beta\, Cos\theta\, Sin\alpha_T \end{vmatrix} \qquad \text{...... (2.17)}$$

The moment at the blade root for a differential element due to gravity can be obtained as

$$\begin{bmatrix} dM_{xG_3} \\ dM_{yG_3} \\ dM_{zG_3} \end{bmatrix} = \begin{bmatrix} r_3\, dF_{y_3} \\ -r_3\, dF_{x_3} \\ 0 \end{bmatrix} \qquad \text{..... ....(2.18)}$$

The equation for total moments due to gravity force in different directions in S_3 coordinate system for one blade with respect to blade attachment point can be expressed as follows :

Flapwise moment, $$M_{xG_3} = \int_0^R r_3 dF_{yG_3} \qquad \text{..... ....(2.19)}$$

Edgewise moment, $$M_{yG_3} = \int_0^R r_3\, dF_{xG_3} \qquad \text{..................... ....(2.20)}$$

2.4 Centrifugal Force

The centrifugal force in the blade of a horizontal axis wind turbine will create tensile and bending stresses in the blade. The root of the blade must be strong enough to resist the tensile and bending loads.

Centrifugal force can be written in local S_3 coordinate system for a differential element as

$$\bar{dF}_{c_3} = \left[k_\beta\right]^T \left[\bar{dF}_{c_2}\right]^T \qquad \ldots\ldots \qquad \ldots\ldots\ldots\ldots \qquad \ldots\ldots(2.21)$$

$$\text{where } \bar{dF}_{c_2} = dm\,(\Omega^* \;\Omega^* \;r_{s_2})$$

For a differential element this can be written as

$$\begin{bmatrix} dF_{xc_3} \\ dF_{yc_3} \\ dF_{zc_3} \end{bmatrix} = \begin{bmatrix} 0 \\ -\,dm\,\Omega^2 r_3 \ \mathrm{Cos}\beta\,\mathrm{Sin}\beta \\ -dm\,\Omega^2 r_3 \ \mathrm{Cos}^2\beta \end{bmatrix} \qquad \ldots\ldots \qquad \ldots\ldots\ldots\ldots \qquad \ldots\ldots(2.22)$$

At the blade attachement point the moment for a differential element can be expressed as

$$\bar{dM}_{c_3} = \bar{dF}_{c_3} * \bar{dF}_{s_3} \qquad \ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots \qquad \ldots\ldots (2.23)$$

This leads to the following equation

$$\begin{bmatrix} dM_{xc_3} \\ dM_{yc_3} \\ dM_{zc_3} \end{bmatrix} = \begin{bmatrix} i_3 & j_3 & k_3 \\ 0 & dF_{yc_3} & dF_{zc_3} \\ 0 & 0 & r_3 \end{bmatrix} \qquad \ldots\ldots\ldots \ \ldots\ldots (2.24)$$

Equation (2.24) can be reduced to

$$\begin{bmatrix} dM_{xc_3} \\ dM_{yc_3} \\ dM_{zc_3} \end{bmatrix} = \begin{bmatrix} r_3\ dF_{yc_3} \\ 0 \\ 0 \end{bmatrix} \qquad \ldots\ldots\ldots \ \ldots\ldots (2.25)$$

Now the equation of total moments in different directions for one blade can be obtained as follows:

Around x_3 axis,

$$M_{xc_3} = \int_0^R \left(m\Omega^2 r_3 \ \mathrm{Cos}\beta\,\mathrm{Sin}\beta\right) r_3\ dr_3$$

and

$$M_{yc_3} = 0$$

$$M_{zc_3} = 0$$

3. BLADE NATURAL FREQUENCIES

The natural frequencies of the blade are determined considering the method of assumed modes and using the Lagrange's equation. The blades are assumed to be rotating at a constant angular frequecy Ω. It is also considered that the vibratory blades does not dissipate energy i.e. its maximum potential energy equals maximum kinetic energy.

Considering the lagging plane the equation of natural frequency is given by

$$\sum_{j=1}^{n} [B_{ij} - (l_j^2 + \Omega^2) A_{ij}] C_j = 0 \quad \text{.............................} \quad (3.1)$$

where l_j = ratio of the blade natural frequency to the shaft rotational frequency for the jth mode shape.

For the flapping plane the equation of natural frequency is given by

$$\sum_{j=1}^{n} (B_{ij} - l_j^2 A_{ij}) C_j = 0 \quad \text{...} \quad (3.2)$$

In the present analysis, the following four functions [1] are assumed to calculate the mode shape and the natural frequencies of the blade.

$$\gamma_1(x) = x^4 - 4x^3 + 6x^2 \quad \text{.................................} \quad (3.3)$$
$$\gamma_2(x) = 3x^5 - 10x^4 + 10x^3 \quad \text{..........................} \quad (3.4)$$
$$\gamma_3(x) = 2x^6 - 6x^5 + 5x^4 \quad \text{.................................} \quad (3.5)$$
$$\gamma_4(x) = x^{10} - 2.5x^3 + 1.60743x^2 \quad \text{......... ....} \quad (3.6)$$

The accuracy of the mode shapes and frequencies improves with the number of functions chosen.

4. RESULTS AND DISCUSSIONS

The results are presented for a two bladed 47 m downwind turbine with a tip speed ratio 8, constant rpm and variable pitch. Throughout the theoretical studies NACA 4418 airfoil and Prandtl's tip loss correction were used. The results are calculated for a wind power law exponent of 1/6 and for the present analysis the hub height is considered as reference height.

Figure 1 and 2 show the variations of pitching moment and yawing moment for two blades. At zero coning angle wind shear produces a periodic wind load with a frequency of twice per revolution. The increase of coning angle reduces the effective radius of the blade which ultimately reduces the pitching moment and yawing moment.

Figures 3 and 4 show the variations of blade

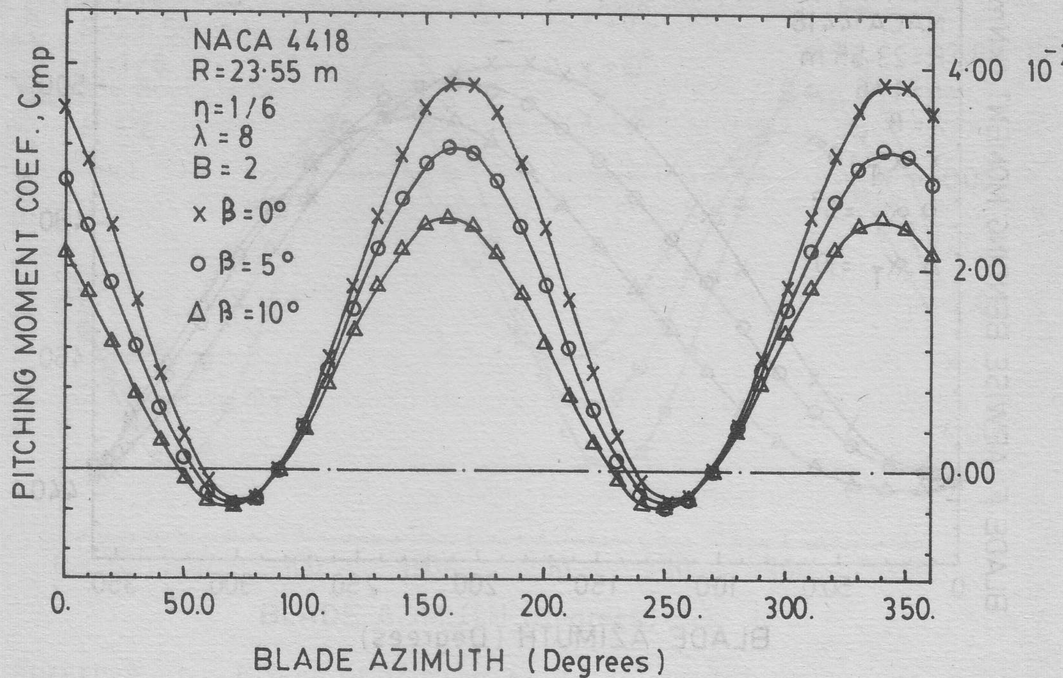

Figure 1 : Effect of coning angle on pitching moment.

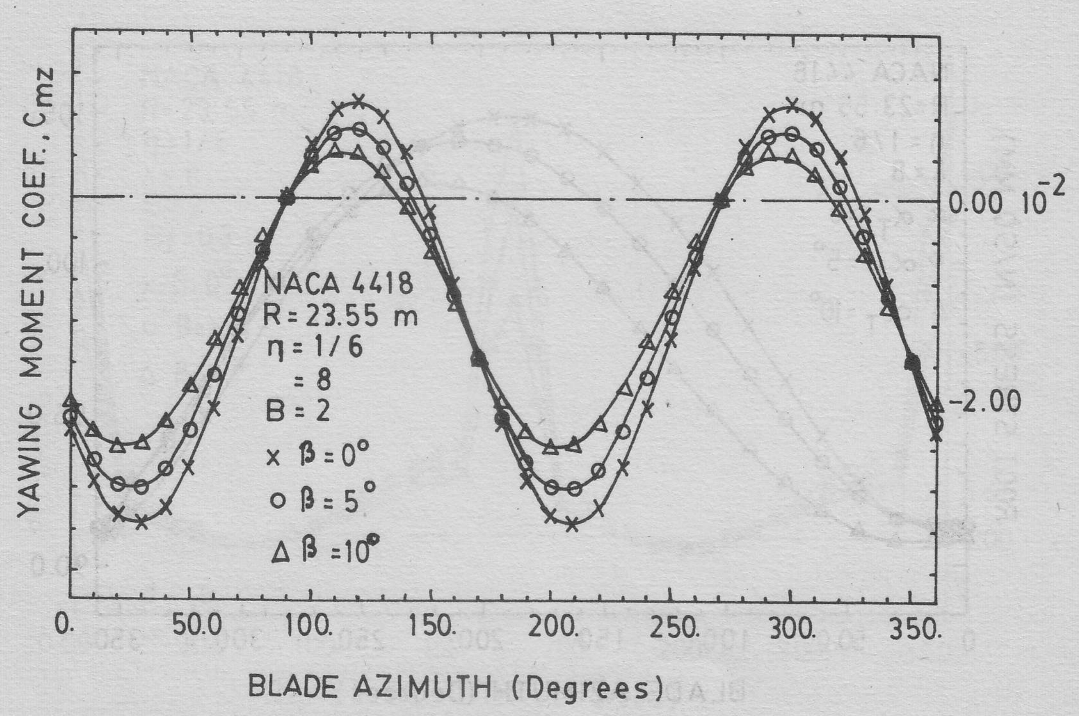

Figure 2 : Variation of yawing moment during one revolution at different coning angles.

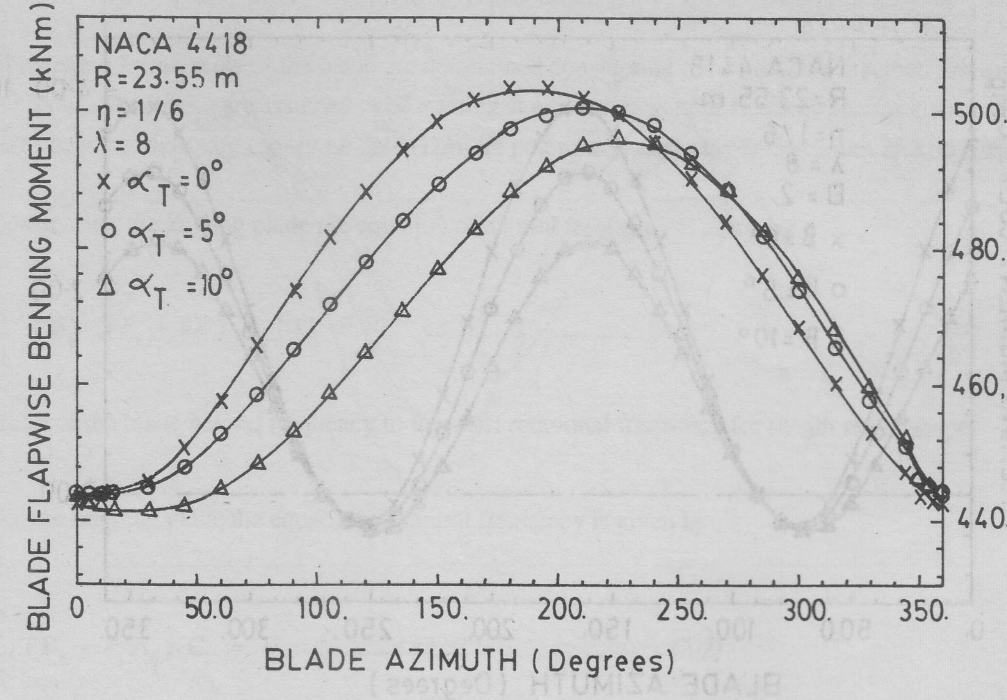

Figure 3 : Blade flapwise bending moment during one revolution at different tilt angles.

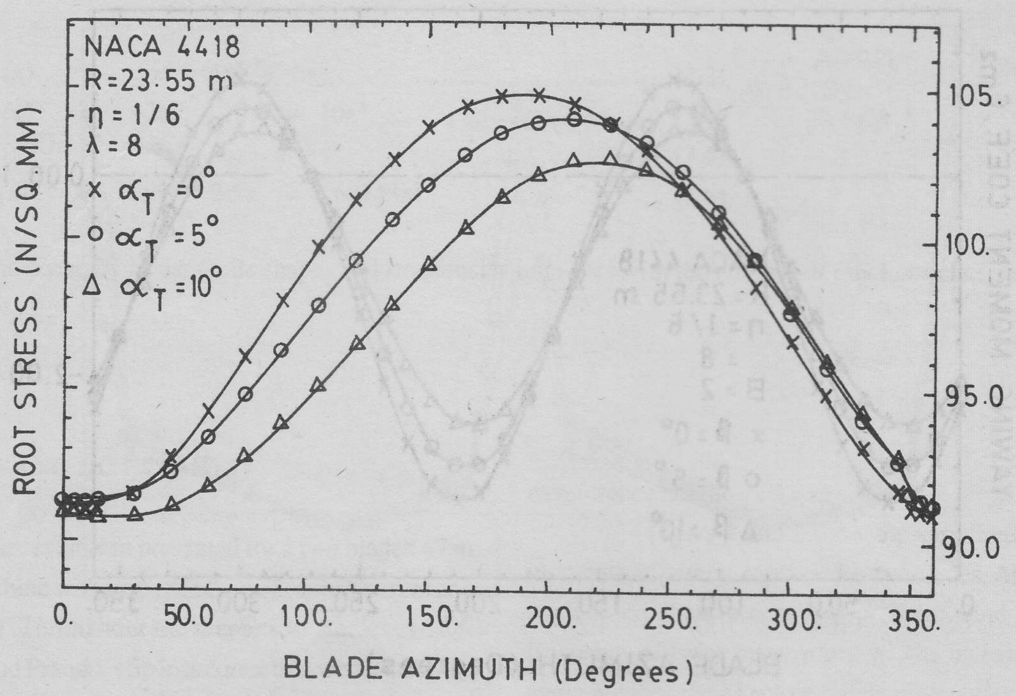

FIGURE 4 : Variation of root stress due to bending at different tilt angles.

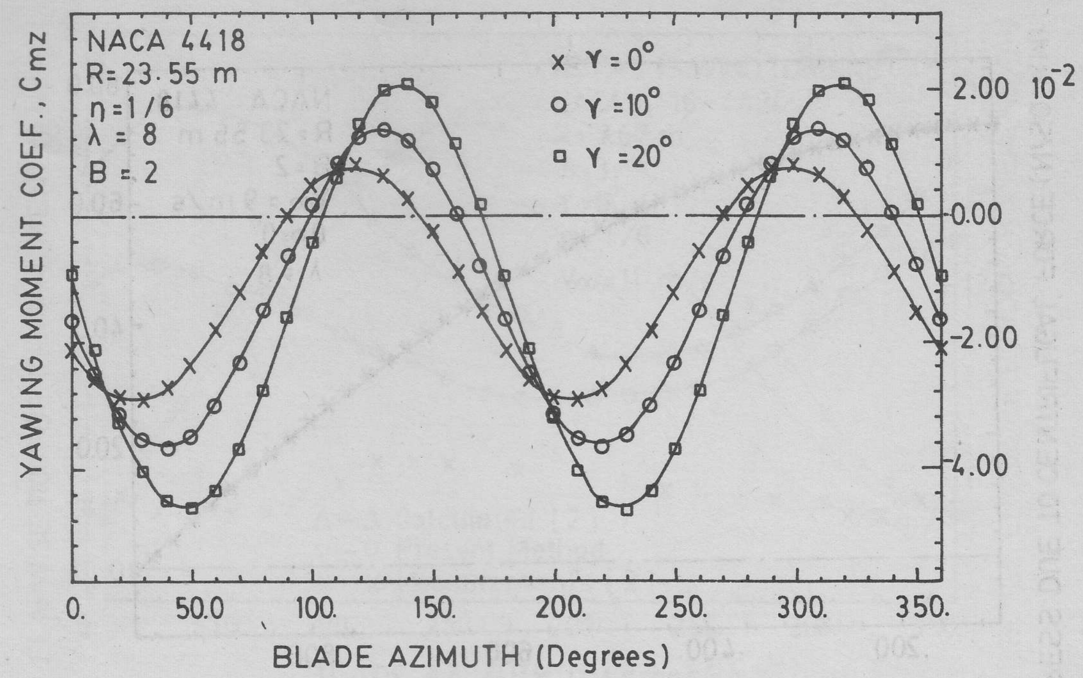

Figure 5 : Effect of yawing angle on yawing moment during one revolution.

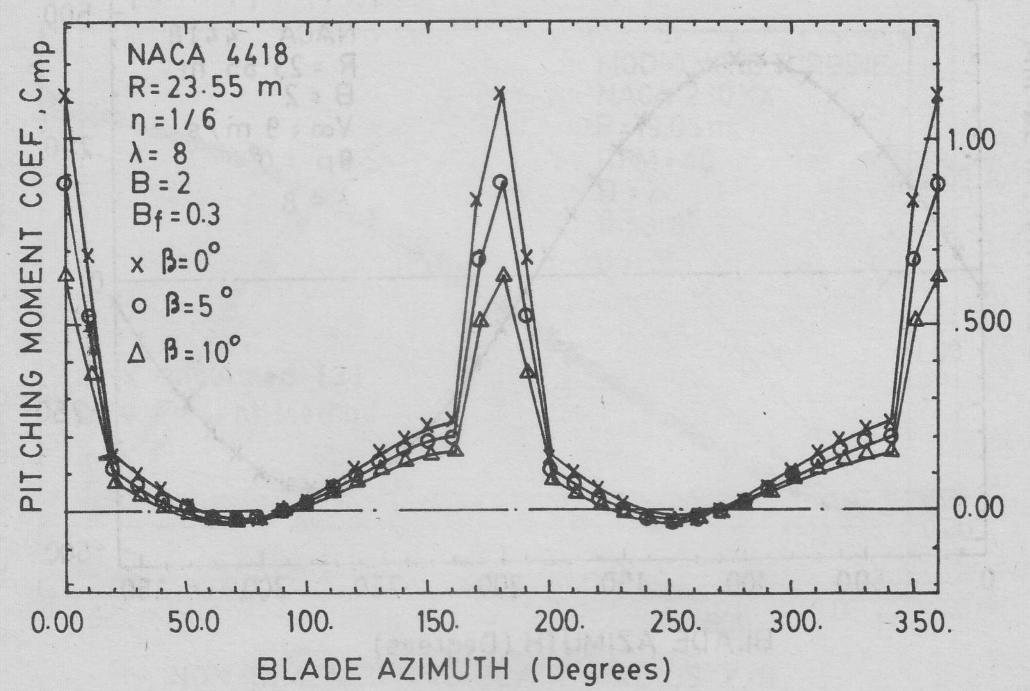

Figure 6 : Effect of tower shadow on pitching moment at different coning angles.

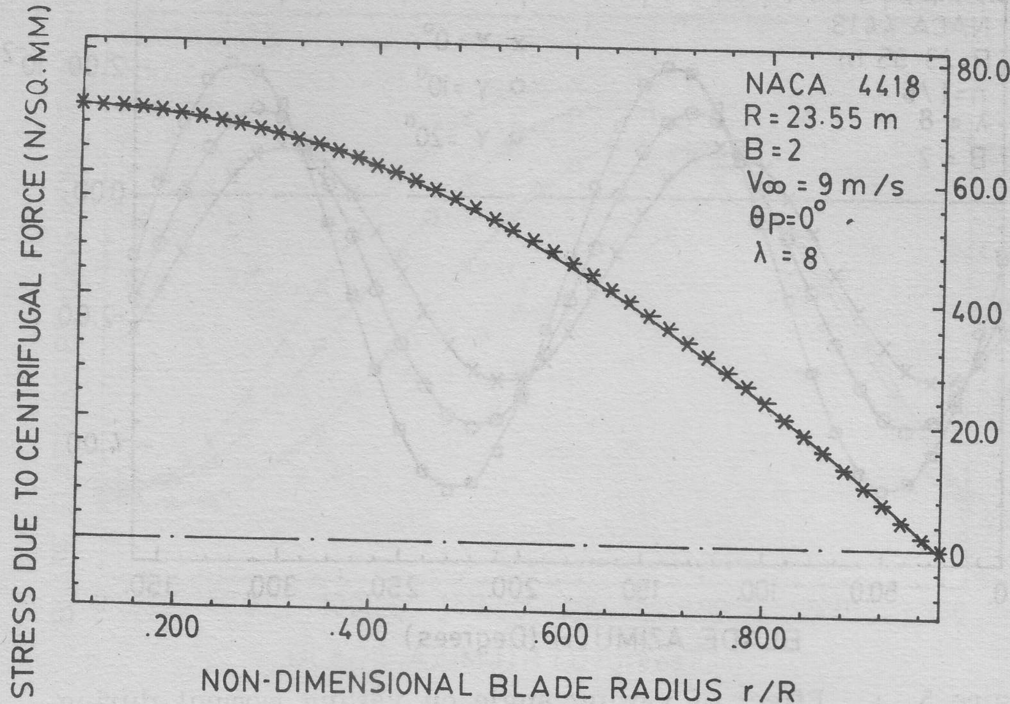

Figure 7 : Stress due to centrifugal force along the length of the blade.

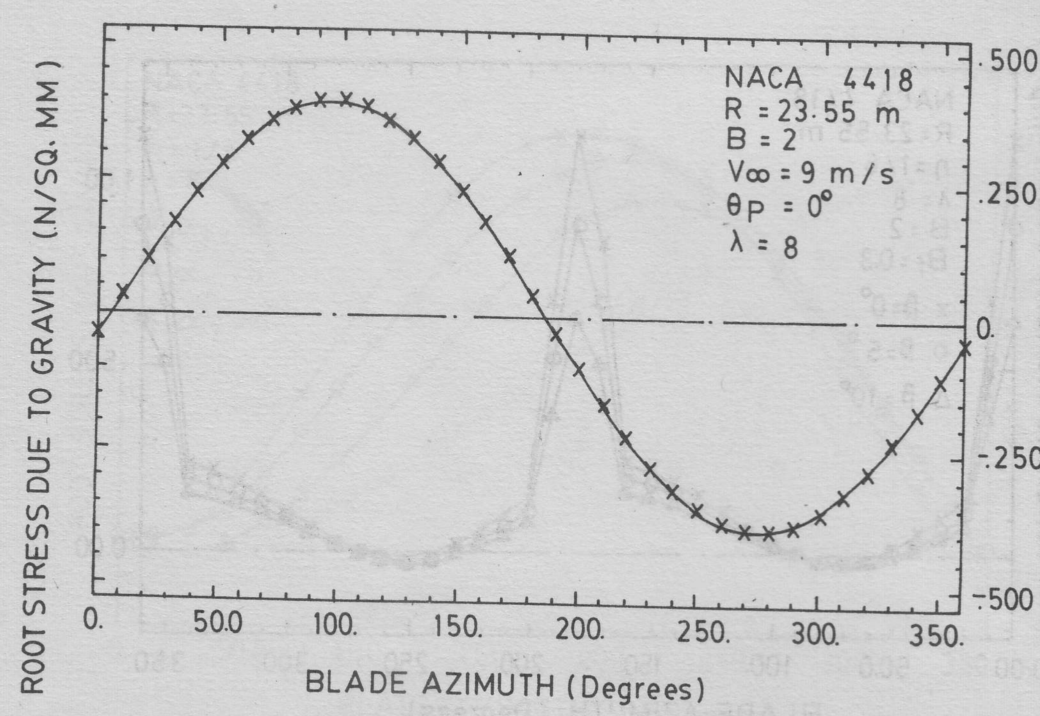

Figure 8 : Cyclic root stress due to gravity.

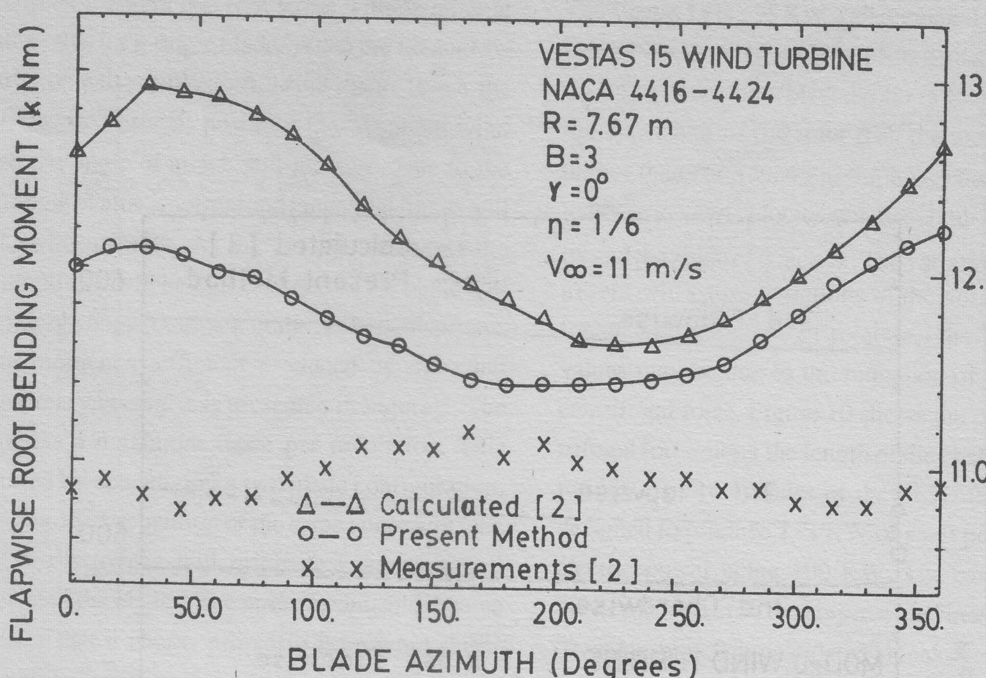

Figure 9 : Comparison of measured and calculated blade flapwise root bending moments due to wind shear.

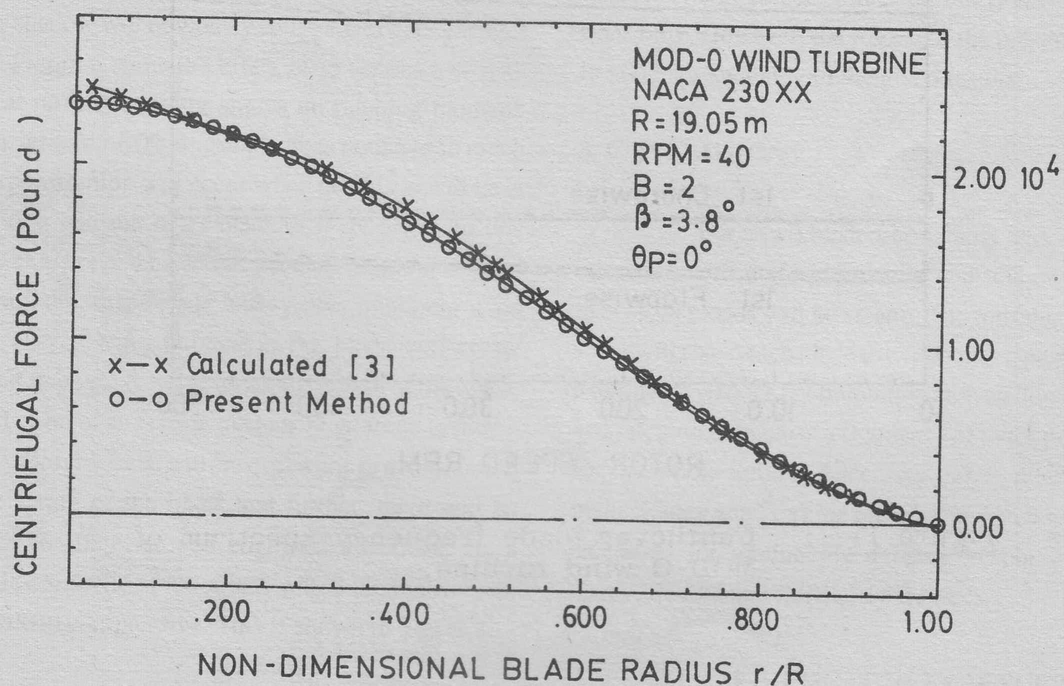

Figure 10 Radial variation of centrifugal force of MOD-O wind turbine blades.

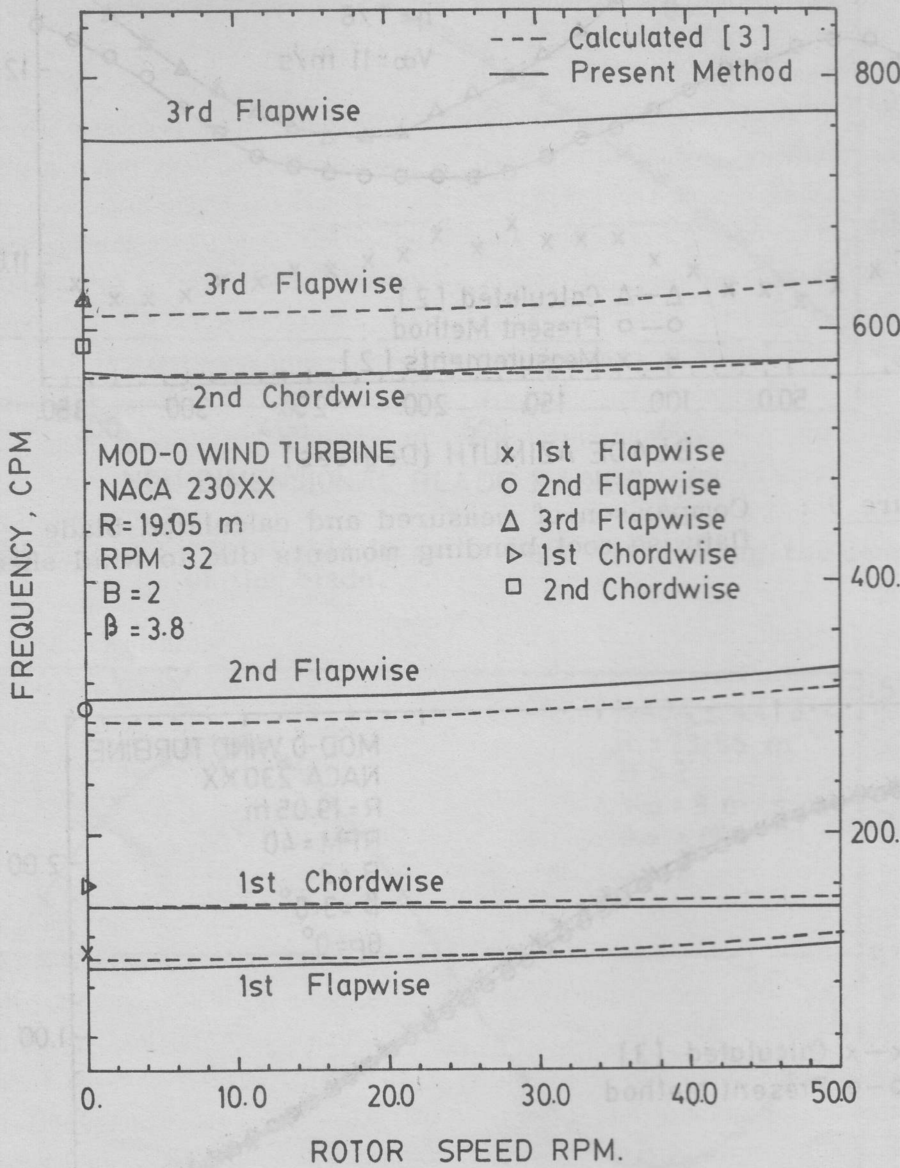

Figure 11 : Cantilever blade frequency spectrum of MOD-O wind turbine.

flapwise bending moment and root stress with azimuth at different tilt angles for a single blade. When the tilt angle is zero the variation will be only due to wind shear. When the blade will be in vertical position the resultant wind velocity and the angle of attack will increase. Due to the variation in angle of attack normal and tangential forces will also change with azimuth. As the tilt agle increases the maximum amplitude will be phase shifted as the blade will be in a favourable angle of attack at the shifted postitions. The yawing moment coefficient produced by the wind turbine at various yaw angles is presented in Figure 5. The variation attains a maximum twice per revolution. This figure is plotted by considering a two blade configuration. So after passing 180°, repetition of the same curve will take place. The two extremes will originate from structural coupling between the blades. The uniform sinusoidal wave shape indicates that it results primarily from wind shear. The aerodynamic interference created by the tower is an important source of periodic wind load. The projected area of the tower structural element, their average drag coefficient, and the sector of the rotor area affected by the wake are necessary for determining this periodic load. The large and abrupt changes that occur as the blade passes through the tower shadow will obviously cause significant changes in blade. Figure 6 show the effect of tower shadow with azimuth at different coning angles on pitching moment. Although variation of coning angle does not have so much effect, large variation will occur when one blade will be at 180° position and the other blade at 0° position. In this condition the upper blade will receive maximum wind velocity and the lower blade will receive minimum wind velocity. The centrifugal force in the blade will create tension and flapwise bending along the length of the blade. The distribution of stress due to centrifugal force is shown in Figure 7. Gravity force will create tension or compression along the length of the blade and further, there will be bending in the flapwise and edgewise directions. As a result, a direct cyclic stress arises which alternates from being tensile to compressive. This is shown in Figure 8.

Figure 9 shows the effect of wind shear on flapwise bending moment of the Vestas 15[2] wind turbine. The Vestas 15 is a 55 KW stall regulated propeller type rotor using NACA 4416-4424 profile series for its blade. The diameter of the three bladed rotor is 15.34 m. For a blade in the upper half of the rotor disk the axial velocity will be higher than for a blade in the lower half of the disk. The increase in wind velocity increases both the resultant velocity and the angle of attack α. The variation in angle of attack in turn will cause veriations in the normal and tangential forces with the angle of rotation. The lowered measured values may be due to the reduction of axial force by the centrifugal force. Figure 10 shows the distribution of centrifugal force along the length of the blade of MOD-0 wind turbine. The two blades of the MOD-0 configuration are designed to produce 133 KW of shaft power, the resulting electric power being 100 KW. The cantilever blade frequency from 0 to 50 rpm is presented in Figure 11 of MOD-0 wind turbine. The calculated values of reference[3] are for the fully coupled flapping and lead-lag bendings. But in the present analysis, flapping and lead-lag bendings are considered separately for simplicity. There are some difference of results for higher modes. The accuracy of the results depends on the number of functions chosen for assumed modes. The measured values are for non-rotating conditions only. However, the results of the present method are in good agreement with the given results.

5. CONCLUSIONS

For a two bladed horizontal axis wind turbine operating with uniform velocity and without any disturbances, the loads will be steady. The introduction of wind shear will cause each blade to experience a periodic force at a frequency of once per revolution and resulting in a yawing and pitching moment at a frequency of twice per revolution for the whole turbine. Successful large, reliable and low maintenance wind turbine must be designed with full consideration for minimizing dynamic response to aerodynamic, inertial and gravitational forces.

APPENDIX - A : LOCAL REFERENCE FRAMES

To calculate the aerodynamic forces acting on the rotor, several co-ordinate systems are introduced in the present analysis. These frame include a reference frame S_o fixed at the top of the tower of the wind turbine with Z_o being the vertical axis and (X_o, Y_o) forming horizontal plane. A second non-rotating frame S_1, fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance and a rotation of tilting angle α_T around the X_o axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally ,a local reference frame S_3 is attached to a particular point of the blade at a distance r from the hub and is rotated over a coning angle b. The relationships between the reference frames can be expressed as

$$S_1 = [K_T]\ So,\ S_2 = [K_\theta]\ S_1,\ S_3 = [K_\beta]\ S_2$$

The transformation matrices are :

$$\text{for tilting angle, } [K_T] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & Cos\alpha_T & -Sin\alpha_T \\ 0 & Sin\alpha_T & Cos\alpha_T \end{vmatrix}$$

$$\text{for azimuth angle, } [K_\theta] = \begin{vmatrix} Cos\theta_k & 0 & Sin\theta_k \\ 0 & 1 & 0 \\ -Sin\theta_k & 0 & Cos\theta_k \end{vmatrix}$$

$$\text{for coning angle, } [K_\beta] = \begin{vmatrix} 1 & 0 & 0 \\ 0 & Cos\beta & -Sin\beta \\ 0 & Sin\beta & Cos\beta \end{vmatrix}$$

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