Mech. Engg. Res. Bull.
Vol. 10, (1987), p p. 27—34

Effect of Blade Shapes on the Performance of A Horizontal Axis Wind Turbine

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Abstract

In this paper perfomance of a horizontal axis wind turbine having different blade shapes are analysed. Three types of blade shapes have been considered : Optimum-chord optimum-twist, linear chord linear-twist and linear-chord zero-twist. It has been observed that a linear-chord linear twist blade is comparable to the optimum designed blade, while offering a considerable reduction in manufacturing time and cost.

Nomenclture

a	axial interference factor	α	angle of attack
a'	tangential interference factor	α_T	tilt angle
B	number of blades	β	coning angle
C	blade chord	β_T	twist angle
C_D	drag coefficient	γ	yawing angle
C_L	lift coefficient	θ_k	blade azimuth angle
C_{L_d}	design lift coefficient	λ	tip speed ratio
F	tip loss factor	λ_d	design tip speed ratio
r	local dadius	λ_r	local tip speed ratio
R	rotor radius	φ	angle of relative wind velocity
V_{∞}	undisturbed Wind velocity	ρ	air density
V_{∞_0}	wind velocity considering shear	σ	solidity
W	relative wind velocity	Ω	angular speed of rotor

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1. Introduction

With the existing blade element theory, a rotor blade can be designed which produces an optimum power coefficient at a given value of the tip speed ratio. A first condition to reach optimum power coefficient is to choose the lift and drag ratio of the profile cosidered and to keep it constant along the entire blade span i.e. the corresponding lift coefficient and the correspon-ding angle of attack has to be taken constant al-ong the blade span. A second condition is that both a strongly varying blade chord length along the span and a strongly varying blade twist are accepted. This leads to a very complicated rotor blade which can be expensive to manufacture and may not have structural integrity. In this paper, it is shown that it is possible to approach the optimum very closely by taking a linearly tapered and linearly twist blade. A NACA 4418 aerofoil section has been used throughout this study.

2. Aerodynamic Design

The parameters that are necessary for the de-sign point of an optimised rotor are the number of blades, the tip speed ratio and the sectional lift and drag raito. For a given set of these pa-rameters the optimum chord and twist distribu-tion have been calculated using the equations of Glauert. To obtain the optimum configuration the blade is divided into a number of radial stat-ions. Four formulas [1] will be used to describe the information about chord and twist angles.

Local design speed : $\lambda_r = \lambda_d \frac{r}{R}$ 2.1

Relation for flow angle : $\varphi = \frac{2}{3} \tan^{-1} \frac{1}{\lambda_r}$ 2.2

Twist angle : $\beta_T = \varphi - \alpha$ 2.3

Chord : $C = \frac{8\pi r\,(1 - \cos\varphi)}{B\, C_{Ld}}$ 2.4

The blade starting torque can be calculated by

$$Q_{st} = {}^1/_2\, \rho\, V^2_\infty\, B \int_r^R C(r) C_L \left[90 - \beta_T(r) \right] r\, dr \quad \text{... ... ... 2.5}$$

3. Linearization of Chord and Twist

The blade configuration for optimum perform-ance requires that the blade chord and the blade twist angle vary continuously and in such a man-ner as to produce maximum power at given tip speed ratio. Such blades are usually difficult to manufacture and may not have structural integrity In order to reduce these problems it is possible to linearize the chord and the twist angle. In con-sidering such linearizations it must be realized that about 75% of the power that is extracted by the rotor from the wind is extracted by the outer half of the blades. This is because the blade sw-ept area varies with the square of the radius and the efficiency of the blades is less at smaller radii where the local tip speed ratio is small. On the other hand, at the tip of the blade the effici ency is low due to the tip losses. For the reasons mentioned above the chord and the twist angle are linearized between r=0.5 R and r=0.9 R. The equations for linearized chord and twist can be written in the following way :

$$C = C_1 r + C_2 \quad \text{.. ... ... — 3.1}$$

$$\beta_T = C_3 r + C_4 \quad \text{... ... ... 3.2}$$

where C_1, C_2, C_3 and C_4 are constants. With the value of C and β_T at 0.5 R and 0,9 R from the ideal blade form the values of C_1, C_2, C_3 and C_4 can be found out. The ultimate expressions for chord and twist of linearized blade can be written as :

$$C = 2.5\,(C_{90} - C_{50}) \frac{r}{R} + 2.25\, C_{50} - 1.25 C_{90} \quad 3.3$$

$$\beta_T = 2.5\,(\beta_{90} - \beta_{50}) \frac{r}{R} + 2.25\, \beta_{50} - 1.25\, \beta_{90} \quad \text{.. 3,4}$$

where

C_{50} = chord of the ideal blade form at 0.5 R

C_{90}=chord of the ideal blade form at 0.9 R

β_{50}=twist angle of the ideal form at 0,5 R

β_{90}=twist angle of the ideal blade form at 0.9 R

A further simplification of the blade shape consists of omitting the twist altogether.

4. Aerodynamic Forces

After a wind turbine rotor is optimally designed, the aerodynamic forces and moments may be calculated. These forces and moments are obtained by applying the blade element and momentum theories. For this purpose, the velocity components of the air flow relative to any point on the blade and also the induced velocity components have to be known. Several reference frames are considered to include the effect of wined shift, tilte, azimuth and coning. This has been discussed in Appendix A. In the fixed reference frame S_o the wind velocity can be expressed as

$$\bar{V}_{so}=V_{\infty}\begin{vmatrix}\cos\nu\\ \sin\nu\\ 0\end{vmatrix} \quad \text{where } \nu \text{ is the yaw angle} \qquad \cdots \ 4.1$$

Expressed in the local co-ordinate S_3 attached to a point on the blade the wind velocity components can be described as

$$\bar{V}_{s_3}=\Big[\,K_{\beta}\,\Big]^T\Big[\,K_{\theta}\,\Big]^T\Big[\,K_T\,\Big]^T\bar{V}_{so} \qquad \cdots \ 4.2$$

Above equation can be written as

$$\bar{V}_{s_3}=\Big[\,k_{\beta}\,\Big]^T\Big[\,K_{\theta}\,\Big]^T\Big[\,K_T\,\Big]^T\begin{vmatrix}\cos\nu\\ \sin\nu\\ 0\end{vmatrix} V_{\infty} \qquad \cdots \quad \cdots \quad 4.3$$

This leads to the folloing expression

$$\bar{V}_{S_3}=\begin{vmatrix}\cos\nu\ \cos\theta_k+\sin\nu\ \sin\theta_k\ \sin\alpha_T\\ \sin\nu\ \cos\alpha_T\ \cos\beta+\cos\nu\ \sin\beta\\ \sin\theta_k-\sin\nu\ \sin\alpha_T\ \cos\theta_k\ \sin\beta\\ -\sin\nu\ \sin\beta\ \cos\alpha_T+\cos\nu\ \sin\theta_k\\ \cos\beta-\sin\nu\ \cos\theta_k\ \sin\alpha_T\ \cos\beta\end{vmatrix} V_{\infty} \qquad \cdots\ 4\,4$$

The rotational motion of the blade will add a velocity component $\Omega r\cos\beta$ in the total velocity vector. Introducing the induced velocity in S_3 co-ordinate system as

$$\bar{v}_{s_3}=\begin{vmatrix}a'\,\Omega\, r\cos\beta\\ V_{\infty}\ a\cos\beta\cos\alpha_T\sin\nu\\ V_{\infty}\ a\sin\beta\cos\alpha_T\sin\gamma\end{vmatrix} \qquad \cdots \quad \cdots \quad \cdots \quad 4.5$$

The components of relative velocity W can be expressed in the local reference system as

$$w_x=V_{\infty_o}\cos\nu\cos\theta_k+V_{\infty_o}\sin\nu\sin\theta_k\sin\alpha_T -\Omega r\cos\beta\,(1+a') \quad \cdots \quad 4.6$$

$$w_y=V_{\infty_o}\Big[\sin\nu\cos\alpha\cos\beta\,(1-a)+\sin\beta\sin\theta_k\cos\nu - \sin\nu\sin\beta\sin\alpha_T\cos\theta_k\Big] \quad \cdots \ \cdots \ 4.7$$

The local angle of attack α is defined as

$$\alpha=\varphi-\beta_T=\tan^{-1}\frac{w_y}{w_x}\quad \beta_T \qquad \cdots \quad \cdots \quad 4.8$$

The velocity components acting on a blade element rotating at a radius r are shown in Figure 1.

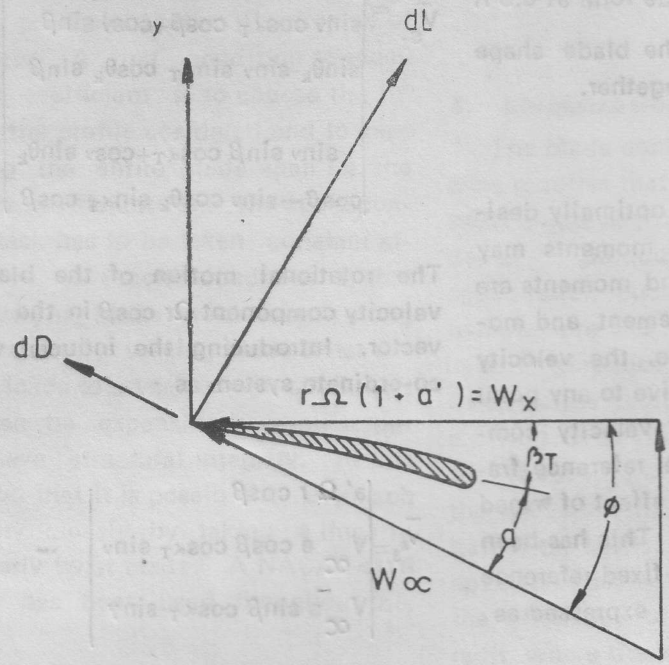


Figure 1 : Blade Flow Velgcity Diagram.

To calculate the interference factors a and a' the expression for the torque and thrust from momentum theory and blade element theoy are equated. From the momentum theory thrust and torque on a radial element can be written as

$$dT = 2\rho r \cos^2\beta \cos^2\alpha_T \sin^2\nu . V^2_{\infty_o}\ a\,(1-a)\, dr.\, d\theta .\, F \quad \cdots \quad 4.9$$

$$dQ = 2\rho r^3 a'\,(1-a)\, V_{\infty_o} \cos\alpha_T \sin\nu \cos^4\beta .\, \Omega .\, dr.\, d\theta .\, F \quad \cdots \quad 4.10$$

Here F is the tip loss factor which is defined as

$$F = \frac{B\Gamma}{\Gamma_\infty}$$

where

Γ = circulation at a radial station r

Γ_∞ = corresponding circulation for a rotor with infinite number of blades.

The tip loss factor has been derived using the method of prandtl [2].

From blade element theory

$$dT = {}^1/_2\,\rho W^2 \cos\varphi\,(C_L + C_D \tan\varphi)\,\frac{Bc \cos\beta .\, dr.\, d\theta}{2\pi} \quad \cdots \quad 4.11$$

$$dQ = {}^1/_2\,\rho W^2 \sin\varphi\,\left(C_L - C_D\frac{1}{\tan\varphi}\right)\frac{Bc\, r \cos\beta\, dr.\, d\theta}{2\pi} \quad \cdots \quad 4.12$$

where $W^2 = W_x{}^2 + {}_yW^2$ 4.13

By equating both sets of equations for thrust and torque the induced velocity factors a and a' may be determined.

$$a(1-aF) = \frac{1}{8}\,\frac{\sigma W^2 \cos\varphi\, c_L}{\cos\beta \cos^2\alpha_T \sin^2\nu\, V^2_{\infty_o}\, F} \quad \cdots \quad 4.14$$

$$a'\,(1-aF) = \frac{1}{8}\,\frac{\sigma W^2 \sin\varphi\, c_L}{r \cos\alpha_T \sin\nu \cos^2\beta\, V_{\infty_o}}\, F \quad \cdots \quad 4.15$$

The drag terms have been omitted in equations (4.14) and (4.15), on the basis that the retarded air due to drag is confined to thin helical sheets in the wake and have little effects on the induced flows [3].

The differential thrust and torque coefficients can be expressed by the following equatious.

$$dC_T = \frac{8}{\pi R^2}(V_{\alpha_0}/V_{\alpha}) \; a F (1-a F) \cos^2\beta \cos^2\alpha_T$$

$$\sin^2\nu (1 + \frac{C_D}{C_L} \tan\varphi) \, r \, dr \, d\theta \; ... \quad 4.16$$

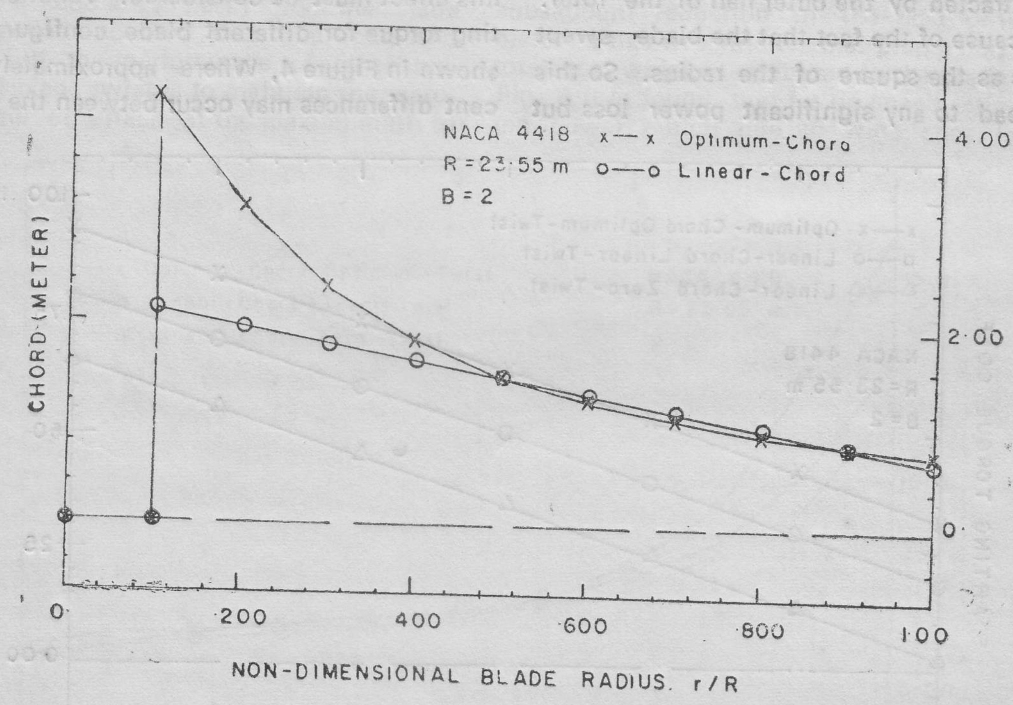
$$dC_Q = \frac{8}{\pi R^3} \cdot \frac{V_{\alpha_0} \, r^3}{V^2_{\alpha}} \; a'F (1-aF) \cos\alpha_T \sin\nu$$

$$\cos^4\beta \;\; 1 - \frac{C_D}{C_L} \frac{1}{\tan\varphi}) \; \Omega . \, dr . \, d\theta \; ... \quad 4.17$$

5. Results and Discussion

The configuration studied is a two-bladed 350 KW downwind wind turbine with a tip speed ratio 8 and variable pitch. Throughout the theoretical studies NACA 4418 airfoil and prandtl's tip loss corection were used to develop the curves. The blade geometry was optimised to give peak performance at 9 m/s. Results of three types of blade shapes have been considered : Optimum-chord optimum-twist, linear-chord linear-twist and linear-chord zero-twist.

Figures 2 and 3 show the distribution of chord and blade setting angles for these three types of blades. From these figures it is found that the changes in chords and twist angles are very small at the outer half of the blade. Large variations with the linear chord and twist distributions are found only at the lower part of the blade. It



Eiqure 2 : Optimum and linearized blade chord distrbution.

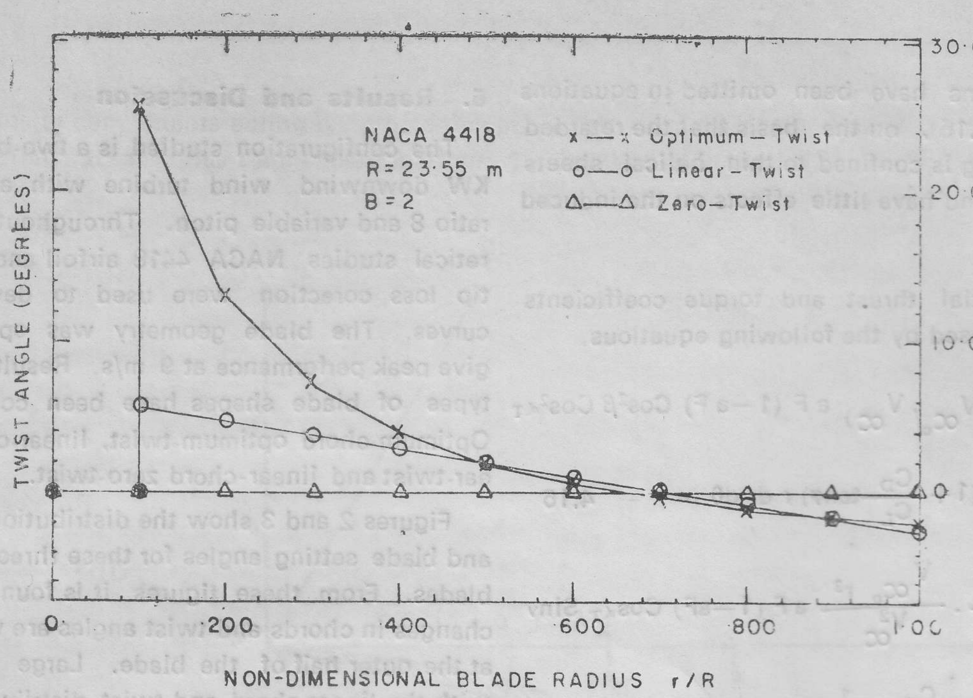


Figure 3 : Optimum and linearized blade twist distribution.

must be realized that about 75 percent of the power that is extracted by the rotor from the wind is extracted by the outer half of the rotor. This is because of the fact that the blade swept area varies as the square of the radius. So this will not lead to any significant power loss but the starting torque will be less and in cases where the starting torque is an important factor, this effect must be considered. Variation of starting torque for different blade configurations is shown in Figure 4, Where approximately 30 percent differences may occur between the optimum

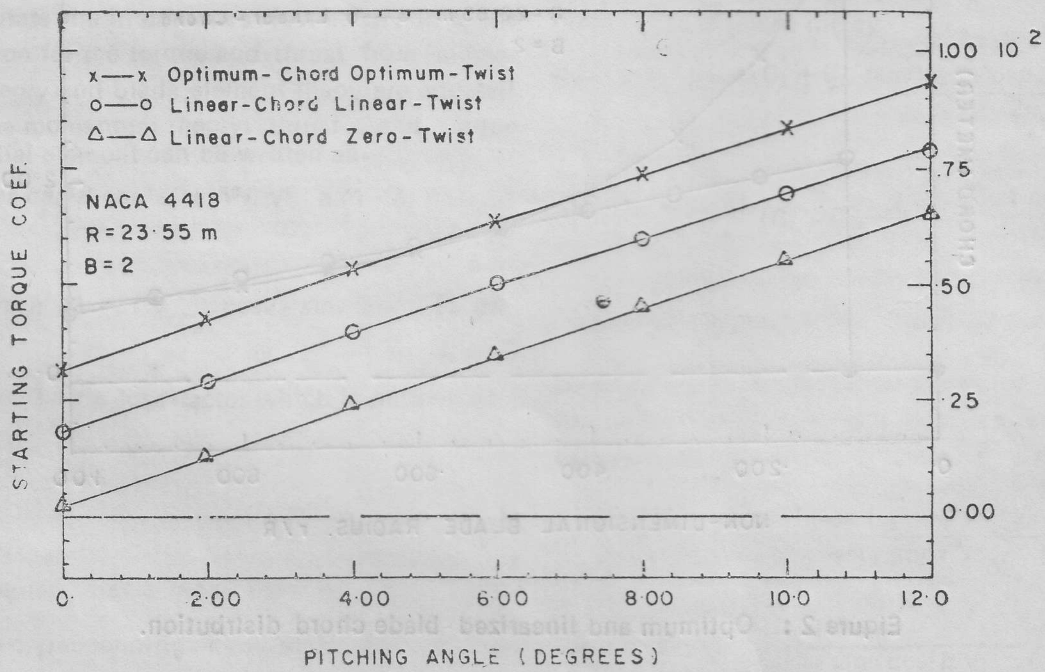
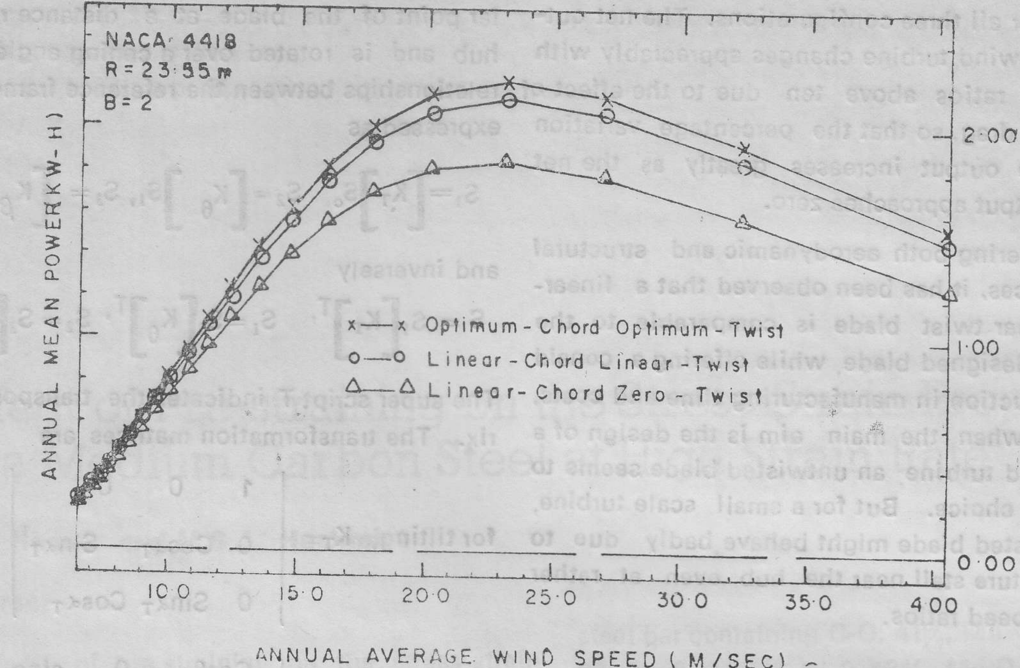


Figure 4 : Comparison of starting torque coefficients for different blade shapes wind turbines.

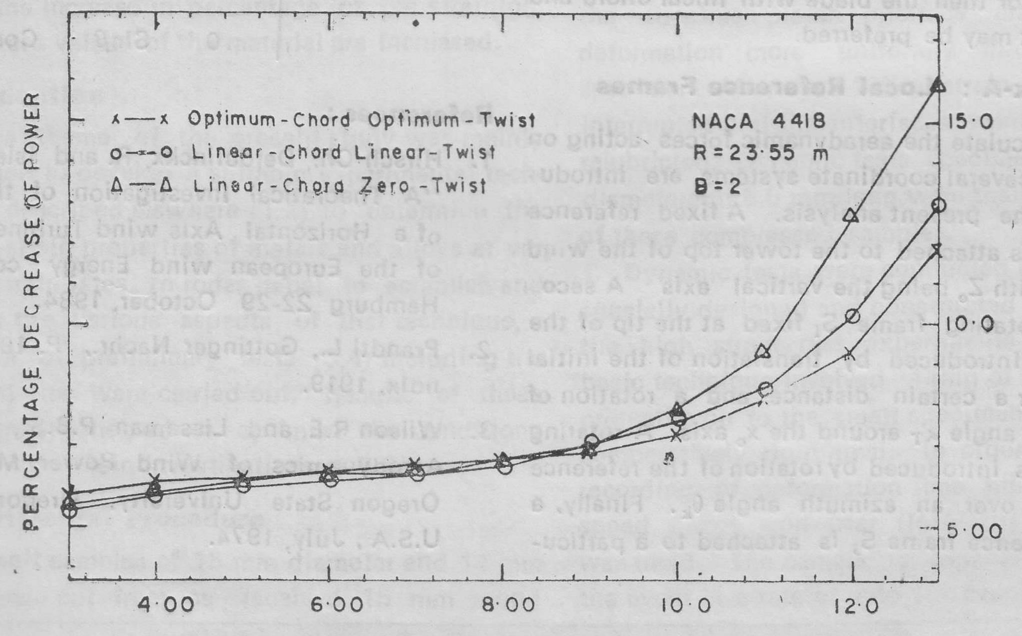
blade and zero-twist blade. The comparison of annual power production is shown in Figure 5.



Fiqnre 5 ; Comparison of annual energy production by different blade shapes wind turbines.

drag ratio. In general terms, the lower power for the untwisted blade arises partly from the blade root being stalled and also from the increased tip losses. But the performance of other two The effect of blade twist is to maintain the aero-dynamic angle of attack at the maximum lift to

types of blades is almost similar. The simpler form of constructions of zero twist might reflect in a reduction in the cost of manufacture. The subsequent reduction in the cost of the complete turbine depends on the proportion of the total cost which is attributed to the blade. From Fig. 6 it is found that for tip speed ratio four to ten power deficit due to wind shear is almost



Fiqqnre 6 : Percent reduction in power output due to wind gradient for different blade shapes wind turbines.

consant for all three configurations. The net output of the wind turbine changes appreciably with tip speed ratios above ten due to the effect of increasing drag, so that the percentage variation in turbine output increases greatly as the net turbine output approaches zero.

Considering both aerodynamic and structural performances, it has been observed that a linear-chord linear twist blade is comparable to the optimum designed blade, while offering a considerable reduction in manufacturing time and costs. However, when the main aim is the design of a cheap wind turbine, an untwisted blade seems to be a good choice. But for a small scale turbine, the untwisted blade might behave badly due to the premature stall near the hub even at rather high tip speed ratios.

6. Conclusions

For the three blade shapes considered, it has been shown that the aerodynamic performance of the three blade shapes are almost similar. Some variations are found for the blade with zero twist. In cases where the starting torque is not an important factor then the blade with linear chord and zero twist may be preferred.

Appendix-A : Local Reference Frames

To calculate the aeradynamic forces acting on the rotor several coordinate systems are introduced in the present analysis. A fixed reference frame S_o is attached to the tower top of the wind turbine with Z_o being the vertical axis A second non-rotating frame S_1 fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance and a rotation of the tilting angle α_T around the x_o axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally, a local reference frame S_3 is attached to a particular point of the blade at a distance r from the hub and is rotated over a coning angle β. The relationships between the reference frames can be expressed as

$$S_1 = \left[K_T\right] S_o, \quad S_2 = \left[K_\theta\right] S_1, \quad S_3 = \left[K_\beta\right] S_2$$

and inversely

$$S = S_1\left[K_T\right]^T, \quad S_1 = S_2\left[K_\theta\right]^T, \quad S_2 = S_3\left[K_\beta\right]^T$$

The super script T indicates the transposed matrix. The transformation matrices are

$$\text{for tilting, } K_T = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}\,\alpha_T & -\mathrm{Sin}\,\alpha_T \\ 0 & \mathrm{Sin}\,\alpha_T & \mathrm{Cos}\,\alpha_T \end{vmatrix}$$

$$\text{for azimuth, } K_\theta = \begin{vmatrix} \mathrm{Coa}\,\theta_k & 0 & \mathrm{sine}_k \\ 0 & 1 & 0 \\ -\mathrm{Sin}\,\theta_k & 0 & \mathrm{Cos}\,\theta_k \end{vmatrix}$$

$$\text{for coning, } K_\beta = \begin{vmatrix} 0 & 0 & 0 \\ 0 & \mathrm{Cos}\,\beta & -\mathrm{Sin}\,\beta \\ 0 & \mathrm{Sin}\,\beta & \mathrm{Cos}\,\beta \end{vmatrix}$$

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