Mech. Engg. Res. Bull.
Vol. 10, (1987), p p. 1—10

Effect of Yaw on Rotor Stability of Horizontal Axis Wind Turbines

Md. Mahbubul Alam*
Md. Quamrul Islam*

ABSTRACT

In this paper a method is presented to study the effect of wind shift angle on the stability of horizontal axis wind turbines. The existing equations of the Modified Strip Theory approach have been extended to include the yaw angle. The equations have been deduced for a downwind horizontal axis wind turbine but these can be equally applied to a upwind rotor with suitable changes of sign.

NOMENCLATURES

F_{x_o}, F_{y_0}, F_{z_0}	forces in x, y and z directions respectively of S_o coordinate system
F_{x_3}, F_{y_3}, F_{z_3}	forces in x, y and z directions respectively of S_3 coordinate system
K_T, K_β, K_θ	transformation matrices for tilting, coning and azimuth respectively
r	local blade radius
S_o	fixed reference coordinate system
S_1, S_2, S_3	coordinate system considering tilt angle, blade azimuth and coning angle respectivety
V_{S_o}, V_{S_3}	wind velocities corresponding to S_o and S_3 coordinate system respectively
V_∞	undisturbed wind velocity
X_o, Y_o, Z_o	distances in X, Y and Z axes of S_0 coordinate system
α_T, β, ν, θ	tilt, coning, yawing, and azimuth angles respectively

INTRODUCTION

The aerodynamic forces on a blade varies during its rotation in the case where the rotor axis is not parallel to the wind direction, even though the wind speed is constant (1). This results from changes in both magnitude and direction of the resulting local wind speed for

**Department of Mechanical Engineering Bangladesh. University of Engineering & Technology, Dhaka.

the profile, which alters with the varying moment of the blade with and against the wind directions. The changes in both wind speed and direction give rise to changes in blade performance with azimuth.

Kottapalli (3) established relationships for stability of horizontal axis wind turbine blades and later Anderson (2) predicted the forces and moments acting on a horizontal axis wind turbine when yawed to undisturbed flow. In reference (4) a procedure for the aerodynamic design and structural analysis of horizontal axis wind turbines is presented. The optimum rotor configuration in determined using the Momentum and the Blade Element Theories and the equations are extened to include various effects.

ANALYSIS

Local Reference Frames

To calculate the aerodynamic forces, on the rotor, several coordinte frames are used in the present analysis. These frames include a reference frame S_0 fixed at the top the tower of the wind turbine with Z_o as the vertical axis and X_o, Y_o axes lying in the horizontal plane. A second frame S_1 fixed at the tip of the nacelle is introduced by translation of the initial frame over a certain distance and a rotation of tilting angle α_T around the X_o axis. A rotating frame S_2 is introduced by rotation of the reference frame S_1 over an azimuth angle θ_k. Finally, a local reference frame S_3 is attached to a particular point on the blade at a distance r from the hub and is rotated over a coning angle β. These are shown

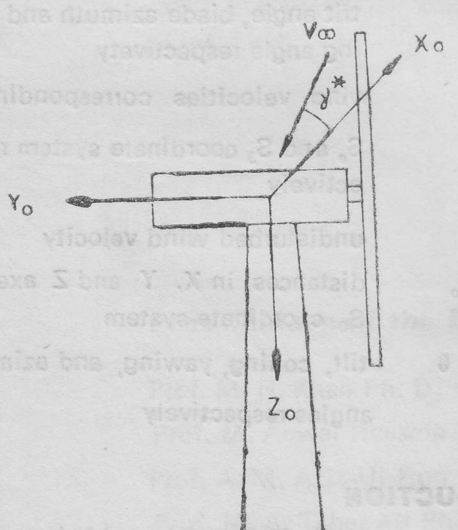

Fig. 1. Coordinate System S_o (Reference Frame fixed at tower top)

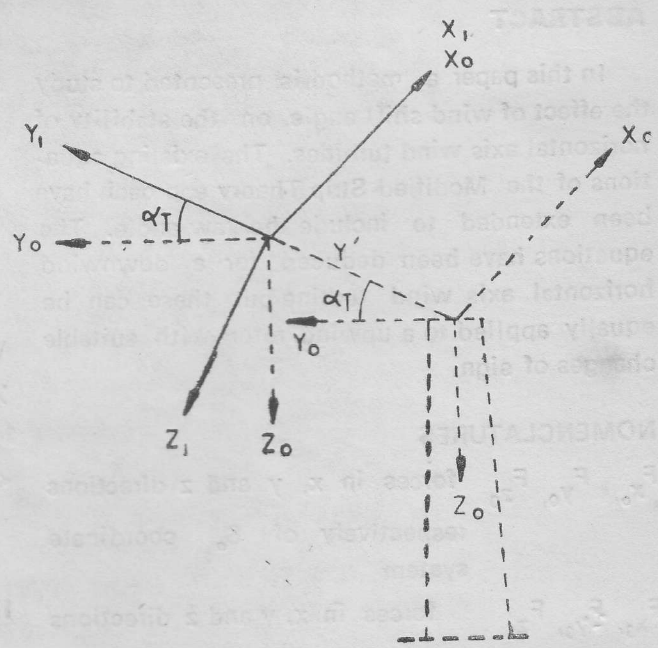

Fig. 2. Coordinate System S_1 (Translaring the Reference Frame over a inistance Y and rotating about x_o by angle T)

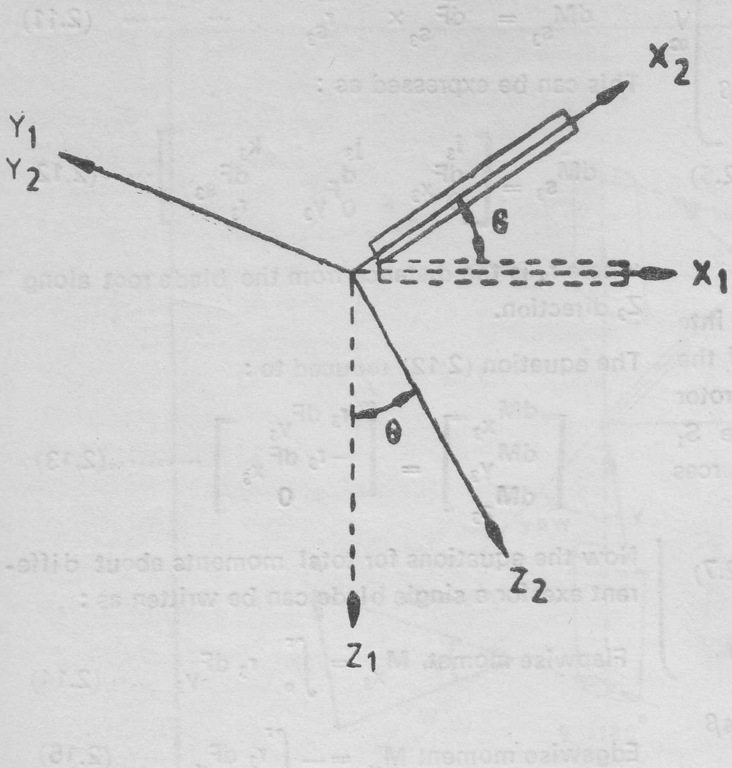

Fig. 3. Coordinate System S_2 (Rotating S_1 about Y_1 by angle θ)

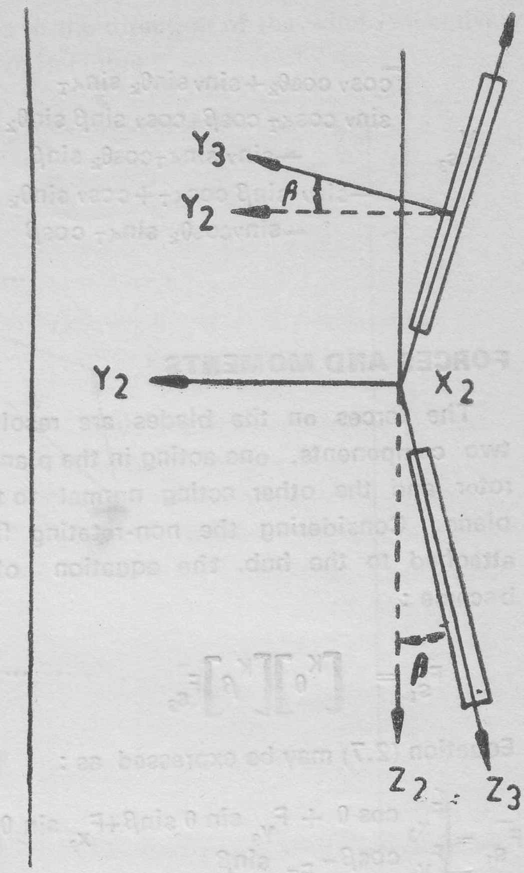

Fig. 4. Coordinate System S_3 (Rotating S_2) about X_2 by angle β)

in Figures 1 to 4. The relationships between the reference frames can be expressed as:

$S_1=[k_T]\ S_o$

$S_2=[K_\theta]\ S_1$

$S_2=[K_\beta]\ S_2$

and inversely

$$S_o=S_1\left[K_T\right]^T$$

$$S_1=S_2\left[K_\theta\right]^T$$

$$S_2=S_3\left[K_\beta\right]^T$$

The superscript T indicates the transpase of a matrix. The transformation matrices are :

$$\text{for tilting, } K_T=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\alpha_T & -\sin\alpha_T\\ 0 & \sin\alpha_T & \cos\alpha_T\end{bmatrix} \quad \ldots\ldots (2.1)$$

$$\text{for azimuth, } K_\theta=\begin{bmatrix}\cos\theta_k & 0 & \sin\theta_k\\ 0 & 1 & 0\\ -\sin\theta_k & 0 & \cos\theta_k\end{bmatrix} \quad \ldots\ \ldots (2.2)$$

$$\text{for coning, } K_\beta=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\beta & -\sin\beta\\ 0 & \sin\beta & \cos\beta\end{bmatrix} \quad \ldots \quad (2.3)$$

In reference frame S_o, considering the wind shift, the wind velocity can be expressed as :

$$\overline{V}_s=V_\infty\begin{bmatrix}\cos\nu\\ \sin\nu\\ 0\end{bmatrix} \quad \ldots\ldots \quad (2.4)$$

$$\text{where } \nu=90^\circ-\nu^* \quad \ldots \quad (2.5)$$

and ν^*=wind shift angle.

Expressed in coordinate system s_3 the wind velocity may be described as :

$$V_{s_3}\begin{bmatrix} \cos\nu\cos\theta_k + \sin\nu\sin\theta_k\sin\alpha_T \\ \sin\nu\cos\alpha_T\cos\beta + \cos\nu\sin\beta\sin\theta_k \\ -\sin\nu\sin\alpha_T\cos\theta_k\sin\beta \\ -\sin\nu\sin\beta\cos\alpha_T + \cos\nu\sin\theta_k\cos\beta \\ -\sin\nu\cos\theta_k\sin\alpha_T\cos\beta \end{bmatrix} V_\infty \quad \ldots\ldots(2.5)$$

FORCES AND MOMENTS

The forces on the blades are resolved into two components, one acting in the plane of the rotor and the other acting normal to the rotor plane. Considering the non-rotating frame S_1 attached to the hub, the equation of forces become :

$$\bar{F}_{s_1} = \left[K_\theta\right]\left[K_\beta\right]\bar{F}_{s_3} \quad \ldots\ \ldots\ (2.7)$$

Equation (2.7) may be expressed as :

$$\bar{F}_{s_1} = \begin{pmatrix} F_{x_3}\cos\theta + F_{y_3}\sin\theta\sin\beta + F_{x_3}\sin\theta\cos\beta \\ F_{y_3}\cos\beta - F_{z_3}\sin\beta \\ -F_{x_3}\sin\theta + F_{y_3}\sin\beta\cos\theta + F_{z_3}\cos\theta\cos\beta \end{pmatrix} \quad \ldots\ldots\ (2.8)$$

The forces on the tower top,

$$\bar{F}_{s_0} = \left[K_T\right]\left[K_\theta\right]\left[K_\beta\right]\bar{F}_{s_3} \quad \ldots\ (2.9)$$

on subtitution of trensformation matriccs,

$$\bar{F}_{s_0} = \begin{pmatrix} F_{x_3}\cos\theta + F_{y_3}\sin\theta\sin\beta + F_{z_3}\sin\theta\cos\beta \\ F_{x_3}\sin\alpha_T\sin\theta + F_{y_3}(\cos\beta\cos\alpha_T - \sin\alpha_T\sin\beta \\ \cos\theta) - F_{x_3}(\cos\alpha_T\sin\beta + \sin\alpha_T\cos\beta\cos\theta) \\ -F_{x_3}\cos\alpha_T\sin\theta + F_{y_3}(\cos\beta\sin\alpha_T + \cos\alpha_T \\ \sin\beta\cos\theta) + F_{z_3}(\cos\alpha_T\cos\beta\cos\theta - \sin\beta\sin\alpha_T \end{pmatrix}$$

In reference trame S_3, the expression for moment for a differential element can be writien :

$$d\bar{M}_{s_3} = d\bar{F}_{s_3} \times \bar{r}_{s_3} \quad \ldots\ \ldots\ (2.11)$$

This can be expressed as :

$$d\bar{M}_{s_3} = \begin{bmatrix} i_3 & j_3 & k_3 \\ dF_{x_3} & dF_{y_3} & dF_{s_3} \\ 0 & 0 & r_3 \end{bmatrix} \ldots\ (2.12)$$

where r_3 is the distance from the blade root along Z_3 direction.

The equation (2.12) reduced to :

$$\begin{bmatrix} dM_{x_3} \\ dM_{y_3} \\ dM_{z_3} \end{bmatrix} = \begin{bmatrix} r_3\, dF_{y_3} \\ -r_3\, dF_{x_3} \\ 0 \end{bmatrix} \ldots\ldots\ (2.13)$$

Now the equations for total moments about diffe-rent axesfor a single blade can be written as :

Flapwise momet, $M_{x_3} = \int_0^r r_3\, dF_{y_3} \quad \ldots\ (2.14)$

Edgewise moment $M_{y_3} = -\int_0^r r_3\, dF_{x_3} \quad \ldots\ (2.15)$

At the tower top, in system s_0, the moment can be cxpressed as :

$$\bar{M}_{s_o} = \bar{F}_{s_o} \times \bar{r}_{s_o} \quad \ldots\ (2.16)$$

yielding.

$$\begin{bmatrix} M_{x_o} \\ M_{y_o} \\ M_{z_o} \end{bmatrix} = \begin{bmatrix} i_o & j_o & k_o \\ F_{x_o} & F_{y_o} & F_{z_o} \\ X_o & Y_o & Z_o \end{bmatrix} \ldots\ \ldots\ (2.17)$$

ANALYSIS OF STABILITY

When the wind turbine rotor axis is not parallel to the directiun of air flow, that is, When a wind shift angle or yaw exists, the aerodynamic forces and moments on the blades will vary during each revolution although the wind turbine is situated in a steady air flow. This is caused by the changes both in maqnitude and direction of the local resulting wind velocity which varies with the cyclic movement of the blade with and opposed to the direction of the wind respectively as shown in Figure 5.

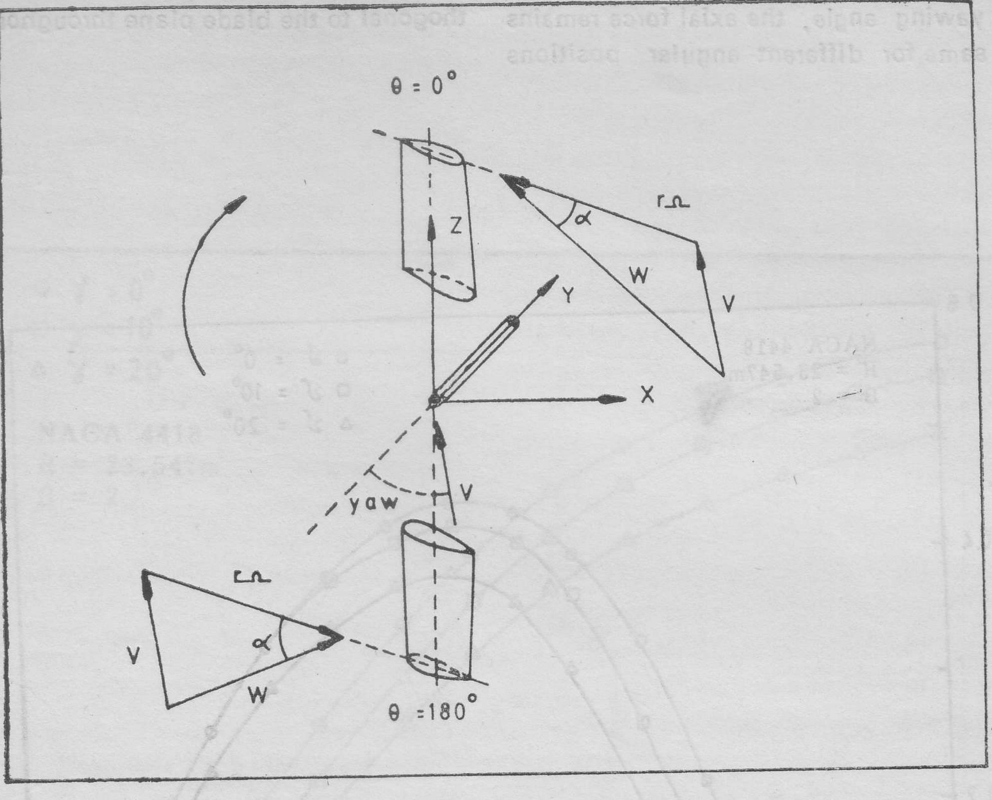

Fig. 5. The Relative wind Velocities at Upper and Lower positions of the Blade in yaw

A horizontal axis wind turbine is said to be in stable state when a disturbance of the equilibrium must create forces and moments within the system that tends to restore the equilibrium. In the following paragraph, expression for tower top forces and yawing moment are presented. For a downwind turbine rotor, without any coning or tilting angle, tower top forces can be expressed as (Appendix A) :

$$\bar{F}_{S_o} = \begin{bmatrix} F_{X_3} \cos\theta + F_{X_3} \sin\theta \\ F_{Y_3} \\ -F_{X_3} \sin\theta + F_{Z_3} \cos\theta \end{bmatrix} \quad (2.18)$$

And equation for yawing moment can be written as (Appendix A) :

RESULT AND DISCUSSIONS

When the rotor axis of a wind turbine is not parallel to the direction of wind flow, the aerodynamic forces and moments vary during a revolution and have a signifcant effect upon various dependant parameters. The power and thrust coefficients produced by the wind turbine yaw at various angles are presented in Figures 6 and 7. From these Figures it can be concluded that the rotor can be yawed for various useful purposes such as to maintain a constant power level when wind speed increases or to unload the rotor for shut-down. Figure shows the distribution of yawing moment as function of the angular position of blade for different yawing angles. As the yawing angle increases, the difference between maximum and minimum values of yawing moment increases in a periodic manner. Because, for zero or lower values of yawing angle, the axial force remains almost the same for different angular positions as in this case wind velocity vectors remain orthogonal to the blade plane throughout.

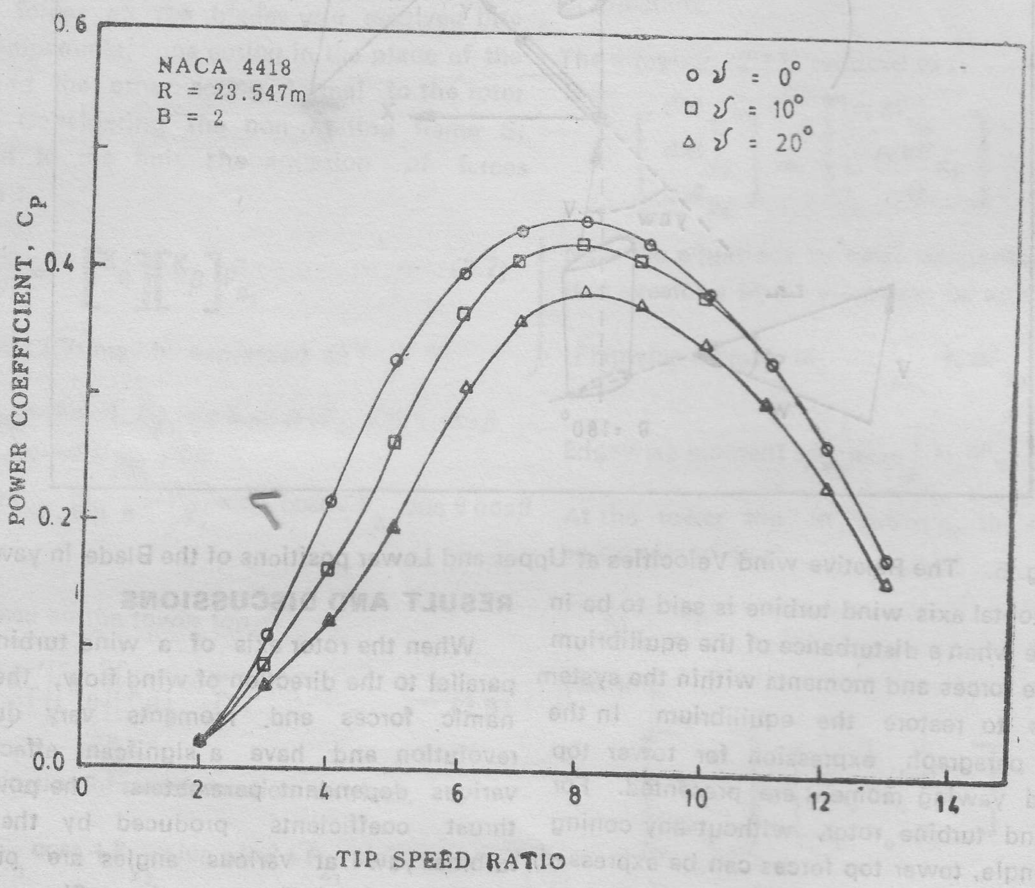

Fig. 6. Variation of power Coefficient with Tip speed Ratio showing the effect of yaw

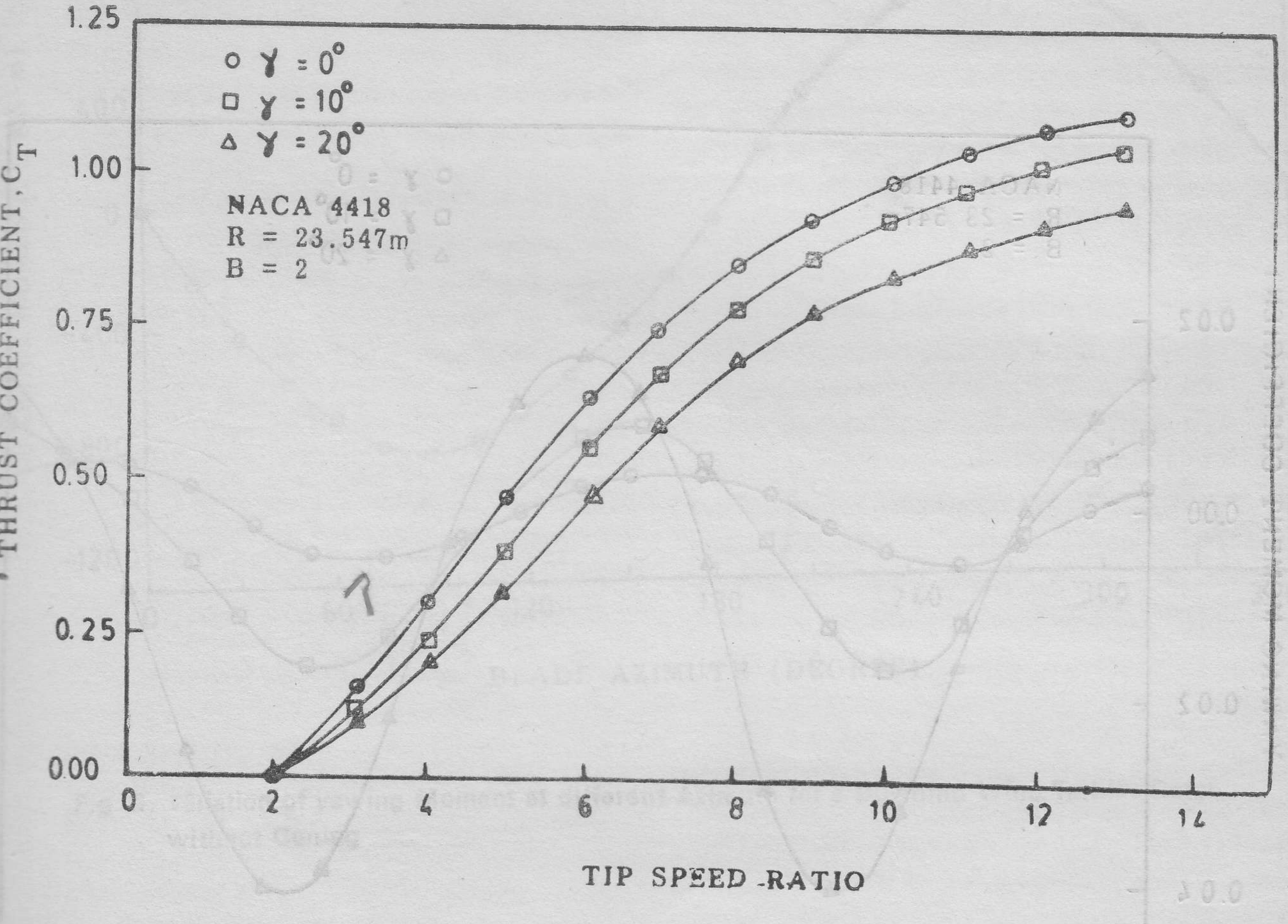

Fig. 7. Variation of Thtust Coefficint with Tip Speed Ratio showing the effect of yaw

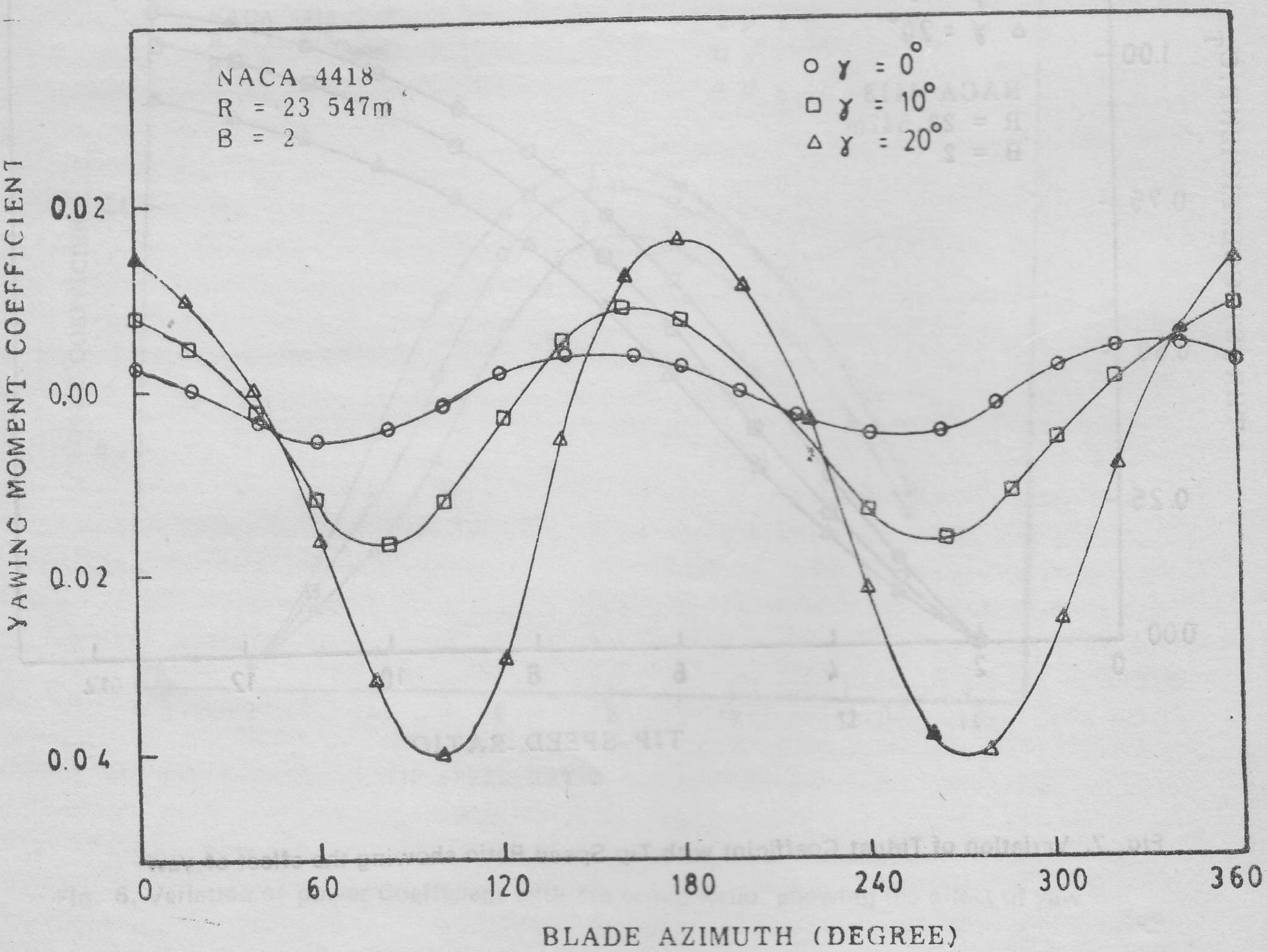

Fig. 8. Effect of yaw on Yawing Moment Coefficnte During one Revolution

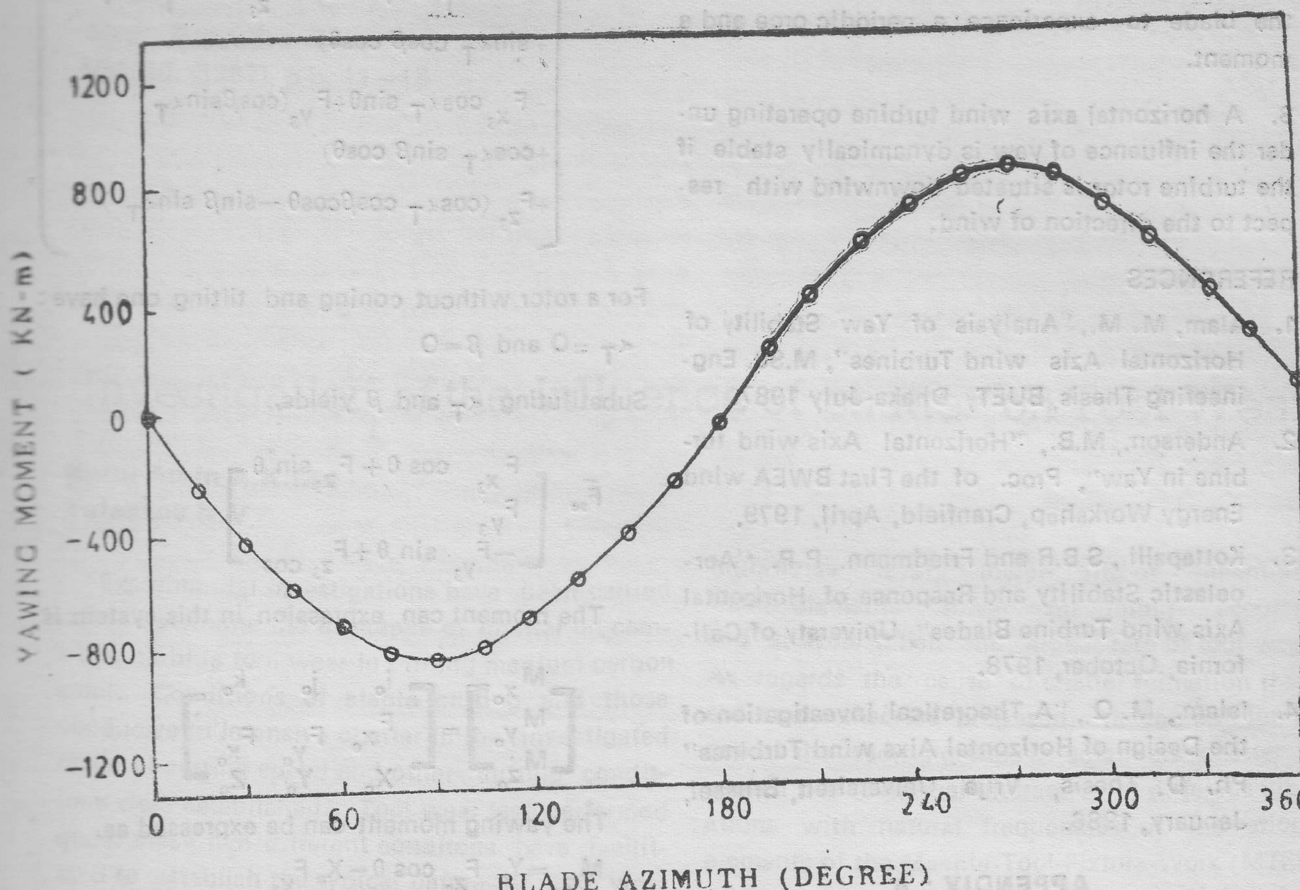

Fig. 9. variation of yawing Moment at different Azimuth for a Downind Wind Turbine Rotor without Coning

For a downwind horizontal axis wind turbine, without any coning or tilting angle, the variation of yawing moment for varicus aizmuthal position is shown in Figure 9. For θ less than 180° the yawing moment is negative. This maens, for a wind turbine rotor without coning or tilting angle yawing moment that developed acts as a restoring torque and the turbine tends to stabilize.

CONCLUSIONS :

1. Yaw has a significant effect on the power and thrust developed by the rotor. Hence effect of yawing angle must be taken into account for the effective design of a horizontal axis wind turbine.

2. For a two bladed horizontal axis wind turbine operating under uniform velocity and without any disturbance, the loads remain steady. However, the introduction of wind shift angle causes the blade to experience a periodic orce and a moment.

3. A horizontal axis wind turbine operating under the influence of yaw is dynamically stable if the turbine rotor is situated downwind with respect to the direction of wind.

REFERENCES

1. Alam, M. M., "Analysis of Yaw Stability of Horizontal Azis wind Turbines", M.Sc. Engineering Thesis, BUET, Dhaka-July 1987.
2. Anderson., M.B., "Horizontal Axis wind turbine in Yaw", Proc. of the First BWEA wind Energy Workshop, Cranfield, April, 1979.
3. Kottapalli., S.B.R and Friedmann., P.P., "Aeroelastic Stability and Response of Horizontal Axis wind Turbine Blades", Universty of California, October, 1978.
4. Islam., M. Q., "A Theoretical Investigation of the Design of Horizontal Aixs wind Turbines" Ph. D. Thesis, Vrije Universiteit, Brussel, January, 1986.

APPENDIX : A

Forces and Moments

In coordinate system S_o, the equation of forces are :

$$\bar{F}_{S_o} = \begin{bmatrix} F_{x_3}\cos\theta + F_{y_3}\sin\theta\sin\beta + F_{z_3}\sin\theta\cos\beta \\ F_{x_3}\sin\alpha_T\sin\theta + F_{y_3}(\cos\beta\cos\alpha_T - \sin\alpha_T\sin\beta\cos\theta) - F_{z_3}(\cos\alpha_T\sin\beta + \sin\alpha_T\cos\beta\cos\theta) \\ -F_{x_3}\cos\alpha_T\sin\theta + F_{y_3}(\cos\beta\sin\alpha_T + \cos\alpha_T\sin\beta\cos\theta) + F_{z_3}(\cos\alpha_T\cos\beta\cos\theta - \sin\beta\sin\alpha_T) \end{bmatrix}$$

For a rotor without coning and tilting one have :

$\alpha_T = 0$ and $\beta = 0$

Substituting α_T and β yields,

$$\bar{F}_{se} \begin{bmatrix} F_{x_3}\cos\theta + F_{z_3}\sin\theta \\ F_{y_3} \\ -F_{y_3}\sin\theta + F_{z_3}\cos \end{bmatrix}$$

The moment can expression in this system is,

$$\begin{bmatrix} M_{x_o} \\ M_{y_o} \\ M_{z_o} \end{bmatrix} = \begin{bmatrix} i_o & j_o & k_o \\ F_{x_o} & F_{y_o} & F_{x_o} \\ X_o & Y_o & Z_o \end{bmatrix}$$

The yawing moment can be expressed as,

$$M_{z_o} = Y_o F_{x_3}\cos\theta - X_o F_{y_3}$$