Experimental investigation of the behaviour of air bubbles in the boundary layer of an inclined tube containing water

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Abstract

Experimental investigation has been done to determine the drag co-efficient of air bubbles and study their behaviour in an inclined tube containing flowing water. It was found that drag coefficient depends on the bubble diameter and the inclination of the tube. The behaviour of the bubble changes at tube angle around thirty degree above which the bubble oscillates, drag coefficient increases and bubble equilibrium Reynolds number decreases.

Nomenclature

A'_b	Cross-sectional area of the bubble.
C_d	Drag coefficient.
D_b	Diameter of the bubble.
D	Diameter of the tube.
F_B	Buoyancy forec.
F_D	Drag force.
g	Acceleration due to gravity.
U	Veolcity.
v	Average velocity in the tube.
v_b	Average velocity of liquid from the bottom to the top of the bubble.
v_{max}	Maximum velocity of liquid at the center of the tube.
V'_b	Volume of the bubble.
y	Any distanee from the tube.
θ	Angle of inclination of the tube.
μ	Viscosity of the liquid.
ρ_L	Density of the liquid.

Introduction

In many industrial appliances cluster of bubbles could be found rising in a liquid column. The motion of these bubbles and the resulting dynamics and other similar flow situations are being studied in two phase flow problems, but very little is known about the behaviour of an individual bubble or a single bubble.

In the present work an experimental model of a single air bubble in an inclined tube containing flowing water has been taken up for visual and photographic study and experimental investigation. The main aims are to find the effect of different tube inclination on the behaviour of the bubble of different diameter and to determine the respective drag coefficient.

Description and theory of the Problem

When an air bubble is inserted in an inclined tube filled with water, it climbs up the tube under the influence of the buoyancy force. If the water is made to flow down the tube, at a certain Reynolds num-

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ber, depending on the inclination of the tube, the drag force acting on the bubble will balance the axial component of the buoyancy force and from the condition of equilibrium the drag coefficient could be determined. Figure 1.1 gives a description of the problem.

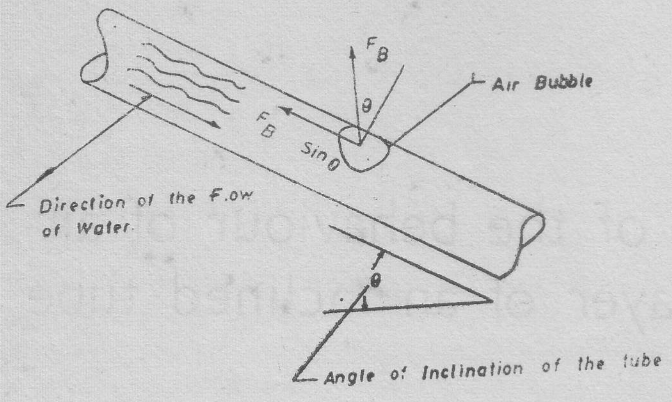


Fig. 1.1: Forces acting on an Air Bubble in an Inclined Tube

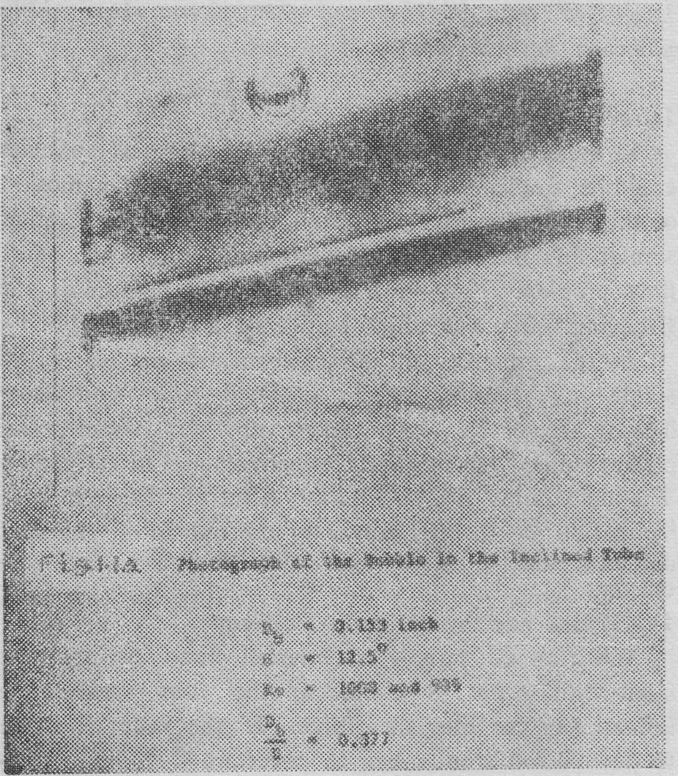


Figure 1.1a

Assumptions

(a) It is evident from the photograph of the bubbles in the inclined tube (Fig, 1,1a), the larger bubbles are nowhere near spherical. They have an oblate shape. In the present study, the bubbles have been considered sperical in shape to minimize the complexity of the mathematical analysis.

(b) The bubbles are found to be separated from the glass tube by a thin layer of water. From a top view of the bubble it was found that there was a circular area of contact between the bubble and the thin layer of water. The contact is not a point contact, this contact perimeter of the bubble may produce some sort of surface tension force depending on the geometry of the bubble at the contact perimeter. It has been assumed that, for the equilibrium condition of the bubble, this surface tension force will be zero. To counteract any error arising from this assumption, the flow condition was varied after the the first reading which altered the equilibrium of the bubble; next, the bubble was again brought to equilibrium by controlling the flow and a second reading was taken and plotted independently.

For the condition of equilibrium of the bubble, the following equation ean be written

$$F_B \sin\theta = C_d\left[\frac{1}{2}\rho_L v_b^2 A'_b\right] \qquad 1$$

where $F_B = \delta_L g V'_b$ is the buoyancy force, and v_b is the average velocity from tha bottom to the top of the bubble. Hence the drag coefficient Cd ean be written as

$$C_d = \frac{\rho_L g V'_b \sin\theta}{\frac{1}{2}\rho_L v_b^2 A'_b} \qquad 2$$

Analysis has been done on the basis of fully developed laminar flow situation, the velceity distribution has been assumed to be parabolic

$$U = \left[1 - \frac{4y^2}{D^2}\right] v_{max}$$

From the above consideration the following equation can be written :

average velocity across the bubble height

$$v_b = 4v\,[D_b/D]\,[1 - D_b/D]$$

bubble Reynolds number $Re_b = \frac{D_b v_b \rho_L}{\mu} = 4R_e\left[\frac{D_b}{D}\right]^2\left[1 - \frac{D_b}{D}\right]$

Reynolds number based on the average velocity in the tube $R_e = \frac{Dv\rho_L}{\mu}$

The dimensional analysis can be made of the drag on the bubble. It is held that evaluating the drag force involves a relationship among the following variables:

$$F_D = \phi\left[v_b^{a_2}, \rho_L^{a_3}, \mu^{a_4}, D_b^{a_5}, D^{a_6}\right]$$

The dimension equation corresponding to above equation is

$$\frac{ML}{T^2} = \phi\left[\left(\frac{L}{T}\right)^{a_2}, \left(\frac{M}{L^3}\right)^{a_3}, \left(\frac{M}{LT}\right)^{a_4}, L^{a_5}, L^{a_6}\right]$$

Eliminating a_2, a_3 and a_5 and collecting the like terms, it can be shown that the drag coefficient is given by

$$C_d = \phi\left[Re_b, \frac{D_b}{D}\right]$$

Apparatus and Procedure

The experimental apparatus is shown schematically in Fig. 1.2. A glass tube 5 ft 6 in long and 0.406 in inside diameter was fitted to a base the inclination of which could be adjusted. The glass tube was connected to a constant head water supply system through a flexible coupling. The water rate could be adjusted by a needle valve. In order that the flow in the test section of the tube be fully developed laminar flow, the test section was located at a distance from the inlet of the tube determined by the equation given by Boussinesq

$$Le = 0.065\ ReD$$

where Le is the entrance length of the tube.

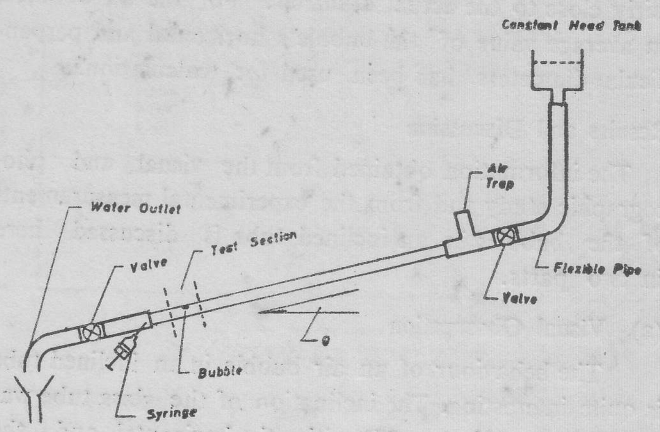
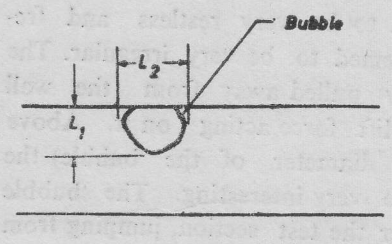


Fig. 1.2: Schematic Diagram of the Experimental Set-up.

After passing through the glass tube, the water was discharged into the atmosphere. To find the mass flow rate of water, the quantity of water was collected in a bucket over a fixed period of time.

Initially, the tube was filled with water and all the valves were closed. Then a hypodermic needle was inserted at the bottom of the glass tube. Before insertion, a bubble was formed in a syringe filled with water. This bubble was pushed through the hypodermic needle into the glass tube. A trial and error method was adopted to obtain a bubble of a particular diameter. A photographic method was employed to determine the volume of the bubble. However, the lens effect was very prominent in the photograph due to the cylindrical boundary of the glass tube. To calibrate the system, solid spheres of standard diameter were introduced in the test section and photographed. It was found from the photographs that the average value of the horizontal and perpendicular diameters (shown in Fig. 1.3) was



$$D_b = \frac{L_1 + L_2}{2} \cdot \frac{1}{M}$$

Fig. 1.3 : Method of determining the Diameter of the Bubble from Photograph. M is the Magnification Factor.

fairly close to the actual diameter. For the air bubbles, an average value of the bubble's horizontal and perpendicular diameters has been used for calculation.

Results aud Discussion

The information obtained from the visual and photographic study and from the experimental measurements of the bubbles in an inclined tube is discussed here in two parts.

(a) Visual Observation

The behaviour of an air bubble in an inclined tube is quite interesting. The inclination of the glass tube was varied from 10° to 87° with the horizontal and the flow of water was adjusted to ensure the equilibrium condition of the bubble. It was found that at low inclinations of the glass tube (below nearly 30°) the bubble, when in equilibrium with the flowing water, was stationary. Above about 30° inclination of the tube (depending on D_b/D) bubble began to oscillate. Figure 1.4 shows the direction of oscillation of the bubble. In this condition it was very difficult to stabilize the air bubble at any section of the tube and the bubble tended to move up or down the tube over a short length. Fine adjustment of the flow of water had to be made continuously to keep the air bubble within the test section. The frequency of oscillation of the bubble was measured by a strobotac and the amplitude of oscillation of the bubble was measured by a cathetometer fitted with a special eye piece. By adjusting the micrometer built in the eye piece the vertical hair line of the eyepiece could be traversed in a horizontal direction.

Figure 1.5 shows the frequency of oscillation and amplitude of oscillation of the bubble as a function of the inclination of the tube. At about an 80° inclination the bubble seemed to be very restless and frequency of oscillation seemed to be very irregular. The bubble was occasionally pulled away from the wall of the tube due to the lift force acting on it. Above 84° (depending on the diameter of the bubble) the bubble behaviour became very interesting. The bubble moves restlessly all over the test section, jumping from one wall to the other, coming to the centre of the stream and again rushing to the wall. This behaviour can best be described as similar to that of a 'live fly' enclosed in a bottle. During this time, the diameter of the bubble was found to decrease. This reduction of the diameter is most probably due to the vigorous agitation of the bubble which forces air to dissolve in the flowing water.

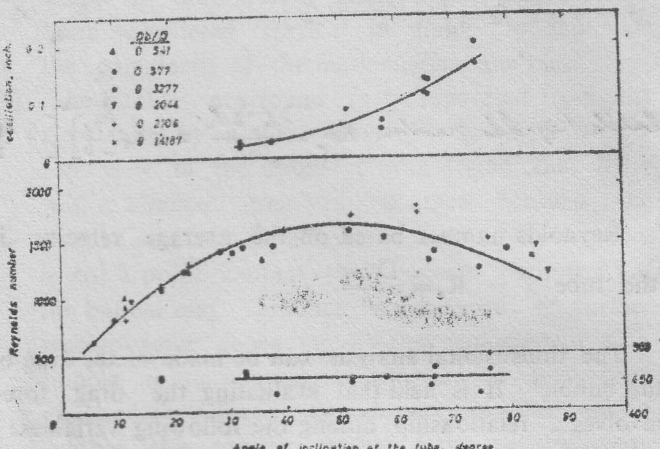
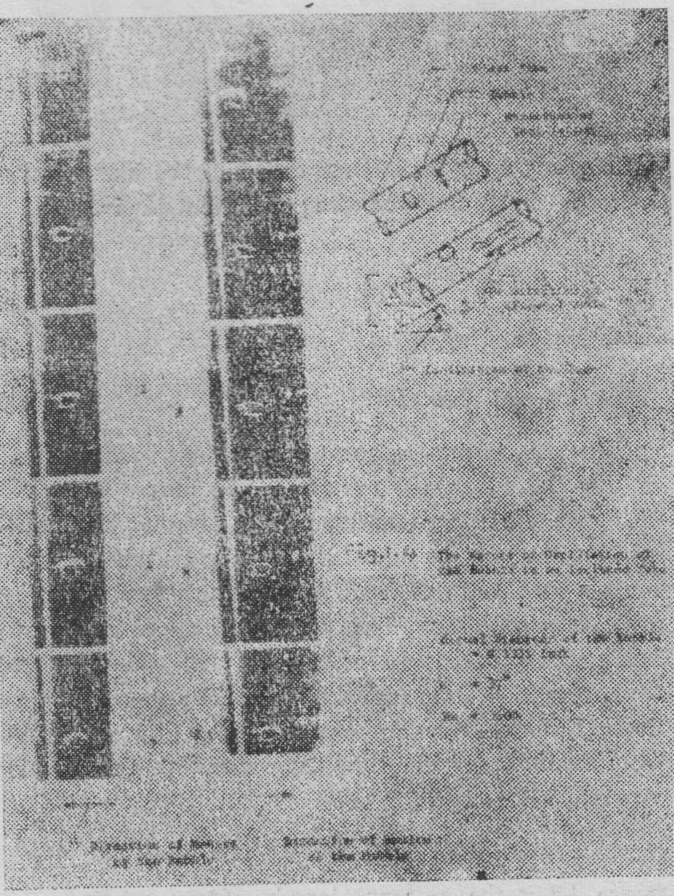


Fig. 1.5. Amplitude of oscillation, frequency of oscillation and Reynolds number are shown as a function of inclination of the tube.



(b) Study of Experimental Measurements

The values of the drag coefficients obtained from Eqn. 2 are plotted as a function of the tube Reynolds number and the bubble Reynolds number in Fig. 1.6 and Fig. 1.7 respectively. In the region of low Reynolds numbers, the drag coefficient increases and at about 30° tube angle the bubble begins to oscillate. Afterwards, as the inclination of the tube is increased, the amplitude of oscillation of the bubble increases and the tube Reynolds number decreases. Consequently, the value of Cd increases. This decrease of Re with increased amplitude of oscillation of the bubble is due to the increased amount of vortices generated due to the oscillation of the bubble. Figure 1.7 . which shows Cd as a function of Re_b , reveals that for higher values of D_b/D oscillation began at higher values of Re_b.

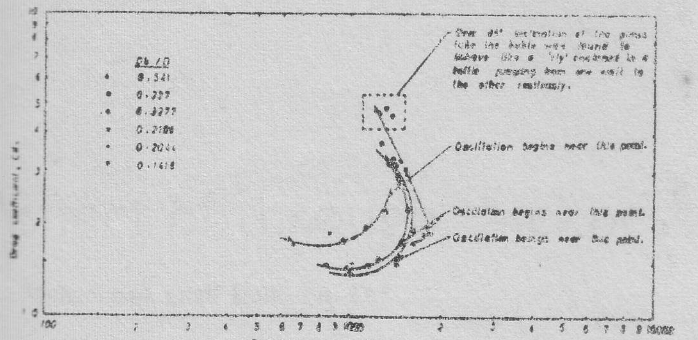


Fig. 1.6 Drag coefficient of bubble in an inclined tube is shown as a function of Reynolds number.

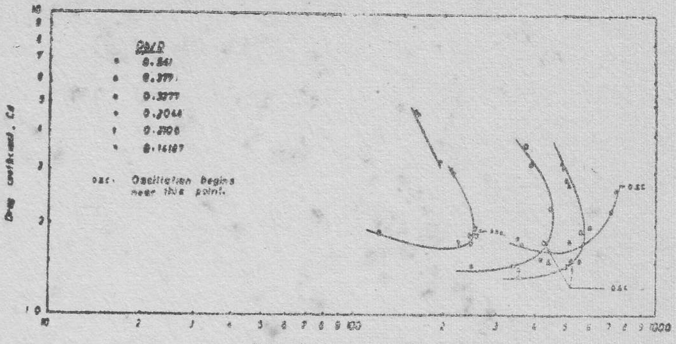


Fig. 1.7 Drag coefficient of bubbles are shown as function of bubble Reynolds number.

The oscillation of the bubble in the tube is a complex problem of gravity, lift and surface tension forces, and vortices that are generated due to flow past a sphere. But why the oscillation begins at above 30° (when its gravity force is 50 percent in the direction of the fluid flow) needs more intensive investigation though an attempt has been made to explain the phenomenon in the following way.

The condition responsible for oscillation of the bubble existed even at the low inclination of the tube but the normal component of the buoyancy force and the friction at the area of contact of the bubble and the tube resisted the motion of the bubble. As the tube inclination increases, this normal force decreases and the oscillation of the bubble sets in. More intensive study specially using flow visualization technique is very necessary to arrive at a definite conclusion about the oscillation of the bubble and to write a mathematical model of the problem.