Design and testing of a two dimensional low speed flutter model

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ABSTRACT

In the present investigation a two-dimensional low speed flutter model (aerofoil section NACA 23012) has been designed. It was fabricated from Indian Teak Wood.

The two-dimensional flutter stability determinant was deduced on the basis of Lagrange's equations for vibration of a flutter system having two degrees of freedom in an air stream. Theodorsen's trial and error method was used to solve the two equations obtained from the flutter stability determinant. The position of the elastic axis was changed for a number of times. For each position of the elastic axis, different values of the reduced velocity (1/k) were assumed and the corresponding roots (x) of the equations were computed. The roots were plotted against the various assumed values of the reduced velocities and two curves were obtained. The point of intersection of the two curves gave the values of 1/k =3.25 and x=1.36. From these two values, the flutter speed was directly calculated to be 94.4 ft/sec. All computatioos were carried out on the Ferranti Sirius Computer. The flutter model was experimentally tested in a subsonic wind tunnel and the flutter speed was observed to be 92.6 ft/sec.

Nomenclature

a	axis location
b	semi-chord
$a.b$	distance between elastic axis and mid-chord point, positive aft of mid-chord.
$(b.x_\alpha)$	distance between c.g. and elastic axis, positive when the c.g. is aft of the elastic axis.
h	distance measured along a direction perpendicular to x.
I_α	mass moment of inertia per unit span about axis x=b.a from mid-chord.
α	angular deflection about elastic axis, positive when the leading edge is up.
j	unit imaginary number.
$k=(b.\omega)/U$	reduced frequency
$1/k=U/(b\omega)$	reduced velocity
K_h	equivalent spring constant in bending per unit span
K_α	equivalent spring constant in torsion per unit span.
m	mass per unit span
M	total mass

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$\mu = m/(\pi \rho b^2)$	density ratio
ρ	density of air at sea level
r_α	dimensionless radius of gyration, defined as $r_\alpha = \sqrt{I_\alpha/(mb^2)}$
$S_\alpha = m.b.x_\alpha$	static mass moment per unit span about axis x=ba, positive when c.g. is aft.
U	the approach velocity
U_F	the flutter velocity
U_D	the divergence speed
$x_\alpha = S_\alpha/(mb)$	the dimensionless static unbalance
$X = (\omega_\alpha/\omega)^2$	where ω is defined as $h = \bar{h}_o e^{iwt}$ and $\alpha = \bar{\alpha}_o . e^{iwt}$
$\omega_\alpha = \sqrt{K_\alpha/I_\alpha}$	uncoupled natural torsional frequency
$\omega_h = \sqrt{K_h}/m$	uncoupled natural bending frequency.

Introduction:

The flutter phenomenon is an aeroelastic, self-sustained excitation in which the external source of energy is the airstream. The air stream feeds energy into the system by virtue of its position or configuration as it is dissipated rapidly by damping. Flutter has perhaps the most far-reaching effects of all the aeroelastic phenomena on the design of high speed aircrafts. Modern aircrafts are subjected to many kinds of flutter phenomena. The classical type of flutter is associated with potential flow and usually, but not necessarily, involves the coupling of two or more degrees of freedom. The non-classical type of flutter which has so far been difficult to analyse on a purely theoretical basis, may involve separated flow, periodic break away and reattachment of the flow, stalling conditions, and various time lag effects between the aerodynamic forces and the motion.

A theory of wing load distribution and wing divergence was first presented in 1926 by Reissner [1]. A theory of loss of lateral control and aileron reversal was published six years later by Cox and Pugsley [2]. The mechanism of potential flow flutter was understood sufficiently well for design use by 1935, largely through the early efforts of Glauert [3], Frazer and Duncan [4], Kussner [5] and Theodorsen [6]. Very recently, Binder [7] carried out investigations on the flutter or galloping of certain structures in a fluid stream. Rao & Chopra [8] have developed a low speed flutter model of a typical wing. Some of the latest contributions to this problem are due to Bisphlinghoff [9].

THE TWO DIMENSIONAL FLUTTER STABILITY DETERMINANT AND ITS SOLUTION.

The flutter can be reduced to two basic problems: the mechanical and the aerodynamic. The first involves the consideration of the motion of the entire airplane structure as a continuous vibrating system acted on by external air forces and internal damping. The problem then reduces to one of writing the equation of motion for such a system. The second basic problem is that of determining the nature of the aerodynamic forces involved. These forces are independent of the static forces which maintain the system in an equilibrium position. The oscillatory aerodynamic forces are those which tend to maintain oscillations about the equilibrium position. Only these aerodynamic forces are considered in the derivation of the equations of the oscillatory motions.

In the following approach, instead of actual distributed mass and geometrical properties of the wing, that of the wing per unit span at some representative position is considered. Thus an approximate representation of the flutter condition for a non-uniform wing of finite aspect ratio has been obtained by considering the motion of this representative unit span. This approach is termed as the two dimensional flutter problem. It should also be noted that the actual motion of the system is asssumed to be a combination of fundamental wing torsion i.e. analysis is done for two degree bending and torsion. Figure 1 shows

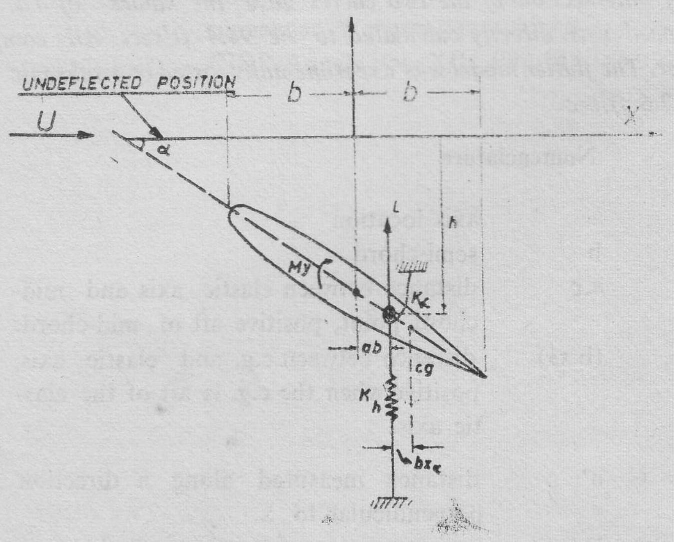

FIG. 1 The Co-ordinate System associated with the aerofoil

the location of the co-ordinate system along with some quantities of primary aerodynamic interest. Lagrange's equation [9] may be used to derive the free vibration equations for this representative section. If the thin aerofoil is subjected to a distribution of pressure difference $(p_u - p_l)$ because of the air flowing past it, we must include generalised external forces in the equations of motion :

$$m\ddot{h} + S_\alpha \ddot{\alpha} + m\omega_h^2 h = Q_h \quad \cdots (1)$$

$$S_\alpha \ddot{h} + I_\alpha \ddot{\alpha} + I_\alpha \omega_\alpha^2 \alpha = Q_\alpha \quad \cdots (2)$$

where the terms on the left hand side are that for mechanical and inertial forces and the terms on the right hand side are that for aerodynamic forces. Q_h and Q_α are the aerodynamic lift & the aerodynamic moment about $x=ba$ as given below:

$$Q_h = \int_{-b}^{+b} (p_u - p_l)\, dx = -L \quad \cdots\cdots\cdots\cdots (3)$$

$$Q_\alpha = \int_{-b}^{+b} (p_u - p_l)(x - ba)\, dx = M_y \quad \cdots\cdots\cdots (4)$$

Here L and M_y are the running lift and the running moment respectively, and are functions of time.

The standard scheme of flutter analysis resembles the one for free vibrations in that we specify simple harmonic motion in advance by setting

$$h = \bar{h}_o . e^{i\omega t} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

$$\alpha = \alpha_o . e^{i(\omega t + \phi)} = \bar{\alpha}_o e^{i\omega t} \quad \cdots (6)$$

This complex representation is justified because the linearity of the equations of motion and the aerodynamic theories to be employed shows that all independent variables in the problem contain time only as the factor $e^{i\omega t}$. We tacitly agree that the actual quantities are always found by taking the real parts of their complex counterparts, recognizing that the algebraic simplification achieved by complex notation outweighs any loss of physical clarity. Since phase shifts in the aerodynamic loads produce a phase difference between h and α, we allow for this by letting one or both of the amplitudes $\bar{h}_o$ and $\bar{\alpha}_o$ be complex numbers. If the time origin is chosen so as to make $\bar{h}_o$ real, the angle by which α leads h is defined in eqn (6) as ϕ, the argument of $\bar{\alpha}_o$.

The assumption of simple harmonic motion changes eqns (1 & 2) to ;

$$-\omega^2 m h - \omega^2 S_\alpha \alpha + \omega_h^2 m h = -L \quad \cdots (7)$$

$$-\omega^2 S_\alpha h - \omega^2 I_\alpha \alpha + \omega_\alpha^2 I_\alpha \alpha = M_y \quad (8)$$

The aerodynamic expressions for L and M_y for low speed flow may be taken from reference (9) as :

$$L = -\pi\rho b^3 \omega^2 \{ L_h \tfrac{h}{b} + [L_\alpha - L_h(\tfrac{1}{2}+a)]\alpha \} \quad (9)$$

$$M_y = \pi\rho b^4 \omega^2 \{ [M_h - L_h(\tfrac{1}{2}+a)]\tfrac{h}{b} + [M_\alpha - (L_\alpha + M_h)(\tfrac{1}{2}+a) + L_h(\tfrac{1}{2}+a)^2]\alpha \quad (10)$$

where L_h, L_α and M_α are functions of the reduced frequency k only, M_h is just $\frac{1}{2}$ for the incompressible case. Substituting eqns (9 & 10) into eqns (7 & 8), and dividing by $\pi\rho b^3\omega^2 e^{iwt}$ and $\pi\rho b^4\omega^2 e^{iwt}$ we get the dimensionless flutter eqns as follows :

$$\frac{\bar{h}_o}{b}\left\{\frac{m}{\pi\rho b^2}\left[1 - \frac{\omega_h^2}{\omega^2}\right] + L_h\right\} + \bar{\alpha}_o\left\{x_\alpha \frac{m}{\pi\rho b^2} + [L_\alpha - L_h(\tfrac{1}{2}+a)]\right\} = 0 \quad \cdots (11)$$

$$\frac{\bar{h}_o}{b}\left\{x_\alpha \cdot \frac{m}{\pi\rho b^2} + [\tfrac{1}{2} - L_h(\tfrac{1}{2}+a)]\right\} + \bar{\alpha}_o\left\{r_\alpha^2 \frac{m}{\pi\rho b^2}\left[1 - \frac{\omega_\alpha^2}{\omega^2}\right] + M_\alpha - (L_\alpha + \tfrac{1}{2})(\tfrac{1}{2}+a) + L_h(\tfrac{1}{2}+a)^2\right\} = 0 \quad \cdots (12)$$

Since eqns (11) & (12) are homogeneous, they constitute an algebraic eigen value with finite solutions occuring at those combinations of speed and frequency for which the characteristic determinant vanishes :

$$\begin{vmatrix} A & B \\ D & E \end{vmatrix} = 0 \quad \cdots\cdots\cdots\cdots\cdots (13)$$

where A,B,D,E are defined as follows:

$$A = \mu\left[1 - \frac{\omega_\alpha^2}{\omega^2} \frac{\omega_h^2}{\omega_\alpha^2}\right] + L_h \quad (14)$$

$$B = \mu x_\alpha + L_\alpha - L_h(\tfrac{1}{2}+a) \quad \cdots (15)$$

$$D = \mu x_\alpha + \tfrac{1}{2} - L_h(\tfrac{1}{2}+a) \quad - (16)$$

$$E = \mu r_\alpha^2\left[1 - \frac{\omega_\alpha^2}{\omega^2}\right] + M_\alpha - (L_\alpha + \tfrac{1}{2})(\tfrac{1}{2}+a) + L_h(\tfrac{1}{2}+a)^2 \quad (17)$$

The determinant given by eqn (13) is known as the two dimensional flutter stability determinant. Theodorsen's method for solving the above determinant may be described as follows. The aerodynamic co-efficients L_h, $L\alpha$, $M\alpha$ etc. in the flutter determinant are complex numbers so that after expansion of the determinant, both real and imaginary parts of the equation should separately vanish. This gives two independent equations. Theodorsen's method of solution is essentially a trial and error method for determining the value of $1/k$ and ω^2 which causes both real and imaginary roots of the equation to vanish simultaneously. If the flutter determinant is expanded and set equal to zero, an equation of the following form is obtained :

$$x^n + C_1x^{n-1} + C_2x^{n-2} + \ldots\ldots + C_n = 0 \ldots \quad (18)$$

where C_j is a complex coefficient which can be expressed as :

$$C_j = R_j + iI_j$$

and the variable $x = (\omega\alpha/\omega)^2$ is a real quantity. Hence the above equation can be expressed as two simultaneous equations of the following form :

$$x^n + R_1x^{n-1} + R_2x^{n-2} + \ldots\ldots + R_n = 0 \quad \ldots\ldots\ldots\ldots \quad (19)$$

$$I_1x^{n-1} + I_2x^{n-2} + \ldots\ldots + I_n = 0 \quad \ldots\ldots\ldots\ldots \quad (20)$$

If x_r is a positive root of eqn (19) and x_i is a positive root of eqn (20) for any assumed value of $1/k$, then by choosing a number of values of $1/k$ and plotting the roots of the equations (19) & (20) vs. $1/k$, two continuous curves of x_r vs $1/k$ and x_i vs $1/k$ are obtained. An intersection of the two curves determines the value of $1/k$ and x for which eqns (19&20) vanish simultaneously. A typical plot is shown in figure 2.

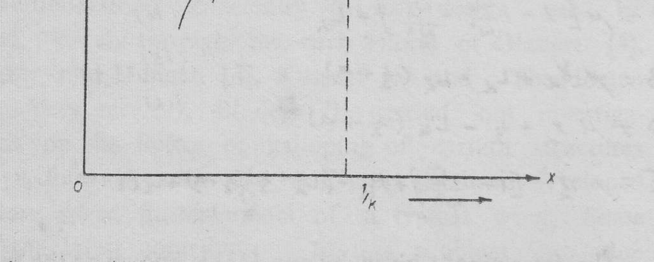

FIG. 2. A typical plot for the Solution of the flutter determinant

Now that the values of x and $1/k$ are found out, one can easily calculate the flutter speed by employing the well known formula (see ref. 9):

$$\frac{U_F}{b\omega} = \frac{1}{k} \quad \ldots \quad \ldots \quad (21)$$

Design of the Model :

Aerofoil Section	:	NACA 23012
Material	;	Indian Teak Wood
Density of Teak Wood	:	48 Lbs/cu. ft.
Chord length of the model	:	C=6 inches
Area of the aerofoil section	:	$0.08075\ C^2$
C.G. from the leading edge of aerofoil	:	0.42C
Area moment of inertia	:	$I_{L.E.} = 186.11 \times 10^{-4} C^4$

Various positions of elastic axis were taken viz., at 27%C, 30%C, 32%C and 35%C from the leading edge, and for each position quantities like μ, $x\alpha$, $r\alpha^2$, $(\frac{1}{2}+a)$ and $(\omega_h/\omega\alpha)^2$ were computed. These were then substituted in the flutter determinant for different (assumed) values of $1/k$ to solve for flutter speed and flutter frequency. It was seen that for the case of elastic axis at 27%C, both the real roots exceed the imaginary root at some value of $1/k$ in between 3.000 & 3.330. From the graph (see figure 3.) at $x=1.36$ and $1/k=3.25$ both the equations are found to be satisfied. Therefore, we have

$$x = 1.36$$
$$= (\omega\alpha/\omega)^2$$

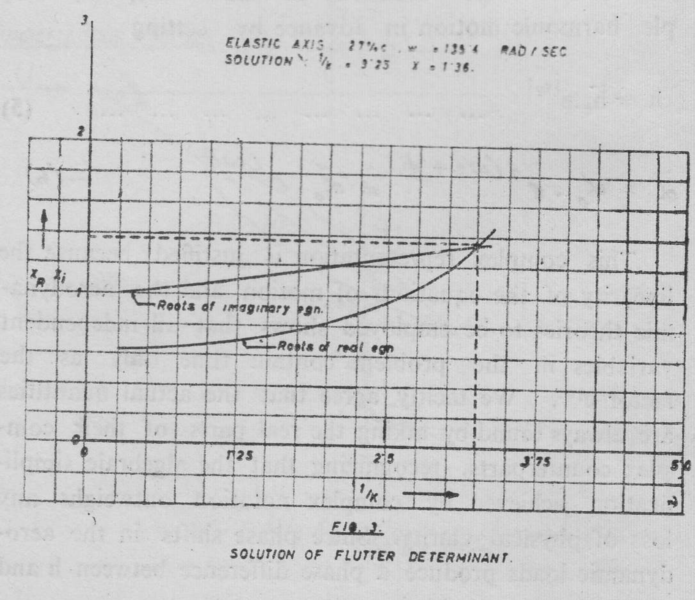

FIG. 3.
SOLUTION OF FLUTTER DETERMINANT.

But for the case of elastic axis at 27%C; $\omega\alpha = 135.4$ rad/sec.

Therefore, $\omega = 116$ rad/sec.

Now, $1/k = U_F/(b\omega) = 3.25$

$\therefore U_F = 94.4$ ft/sec.

The Experiment :

The width of the low speed wind tunnel used for this experiment was two feet. But the span of the model actually tested was one and a half feet. The design essentially being a two dimensional one, to avoid tip effects another enclosure was designed and placed in the side of the test section. Eight springs, four on each side, were attached to the model at the predetermined points and the model was suspended inside the enclosure (figure 4).

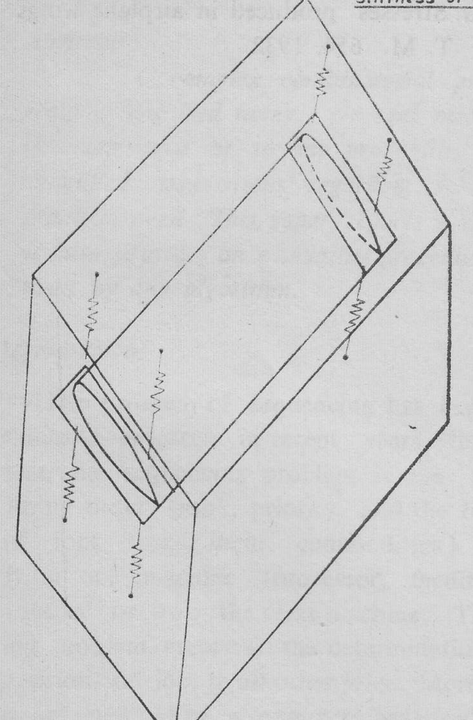

FIG. 4.

A THREE DIMENSIONAL VIEW OF THE EXPERIMENTAL SET UP.

After positioning and suspending the model, the wind tunnel was started. The air speed was gradually increased until the model was blown away. The flutter speed was recorded to be 92.6 ft/sec. The experiment was repeated for a number of times using new springs (having the same spring constant) each time. Almost the same flutter speed was observed every time.

Conclusions

The results obtained from computation and experiment were as follows :

The Computational Result: U_F=Flutter Speed= 94.4 ft/see.

The Experimental Result : U_F= Flutter Speed=92.6 ft/sec.

Hence the design flutter speed agreed quite closely wlth the experimental one.

It must be noted that the flutter speed was computed for a model of span 2 ft. But the model actually tested had a span of 1.5 ft. This loss in mass had been compensated for by the aluminium end-plates (figure 5). Hence the experimental result did not depart widely from the theoretical one.

Figure 5: The experimental set-up

One should note the line of demarcation between the flutter of an aircraft wing in actual flight and the flutter in our experiment. The former represents three dimensional flutter whereas the latter represents a two dimensional flutter. In the former case the wing itself is an elastic structure, but in the present experiment the model as such is a rigid structure. That is why springs were attached to provide elasticsty to the model.

In the actual flutter phenomenon, some special external excitation such as a gust is necessary prior to the onset of oscillations of increasing amplitude. But in the present investigation, flutter was encountered without any external disturbance being given to the system. This can be explained in the light of Kussner theory (10) which shows that at low amplitude the laws of potential flow do not hold because the viscosity of the air is not negligible. Consequently, the aerodynamic forces, for the case of very small oscillations, are smaller than would be expected from potential flow theory, and, therefore, do not induce flutter. Thus due to boundary layer effect, it would take a disturbance of certain minimum value to start flutter.

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