Effect of through thickness strength on bulging process

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Abstract :

Many theories have been put forward to predict the forming process of circular bulges by means of unilateral hydrostatic pressure. Very few of them consider incremental strain theory and anisotropy of the sheet metal. But Yamada and Yokouchi predicted a theory for predicting the bulge forming process using incremental strains and incorporated in it the normal anisotropy of the sheet metal while the equations were simple compared to those of other theories. This theory has been solved numerically in this present work to find the effect of four different values of normal anisotropy R on pressure-polar thickness strain, and polar radius of curvature-polar thickness strain relationships for a given value of strain-hardening index n. For the numerical solution CDC 7600 machine has been used.

Nomenclature

R	Normal anisotropy or strain ratio defined as the ratio of natural width to natural thickness strain in tension tests.
n	Strain hardening index ($\sigma = K\varepsilon^n$)
K	Constant in the stress-strain relationship ($\sigma = K\,\varepsilon^n$)
ε	Equivalent strain
σ	Equivalent stress
ρ_1	Meridional radius of curvature
ρ_2	Circumferential radius of curvature
ε_o	Thickness strain at pole
$\varepsilon\theta$	Circumferential strain
$\varepsilon\phi$	Meridional strain
$\sigma\theta$	Circumferential stress
$\sigma\phi$	Meridional stress
s	Original radius from pole to an element
s′	Current radius from pole to an element
ϕ	Angle with vertical formed by a generic point
p	Fluid pressure, psi
t_o	Original thickness of sheet metal
t	Current thickness of sheet metal
a	Radius of die opening
x, y	Generalized x and y coordinates

dash means derivative with respect to x
dot means rate of change

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Introduction

Most of the equilibrium equations for bulges are similar but the analysis and the complexity of numerical solutions are much dependent on the flow rule and boundary conditions used. Woo [1] used a general method for axisymmetric forming processes based on incremental strain theory but obtained the results on the basis of total strain theory using successive approximations of stresses and strains according to the known stress strain characteristic. Woo pointed out the difficulty of the requirement of high computer time to satisfy the boundary condition if incremental strain theory is used for the numerical solution. Wang and Shamammy [2] investigated the bulge forming processes by incremental and total strain theories but their equations were complicated and not suitable for easy solution. On the other hand Yamada and Yokouchi [3] analysed the problem in a simpler way and suggested a systematic procedure for numerical solution which has been used here for comparing forming processes for materials having different through thickness strength. The solution near the edge of the diaphragm is difficult and inaccurate. The values obtained at the pole for arbitrary materials have been used for the comparisons without quoting the absolute values.

THEORY

For deducing the governing differential equations the following assumptions are made.

i) The material is incompressible.

ii) The sheet metal is isotropic in the plane of the sheet.

iii) There is no friction during the forming process.

iv) The deforming process is considered to be axisymmetric.

Figure 1 shows one stage of hydrostatic deformation of circular sheet metal clamped rigidly at the edge. For this axisymmetric forming process circumferential strain, meridional strain and equilibrium equations can be combined and/or simplified to the following equations.

$$\frac{d(t\sigma_\phi)}{ds} = \frac{t(\sigma_\theta - \sigma_\phi)}{s} e^{(\varepsilon_\phi - \varepsilon_\theta)} \quad (1)$$

and

$$\frac{p}{t} = \frac{2\sigma_\phi \sin\phi}{s\, e^{\varepsilon_\theta}} \quad (2)$$

The strain compatibility equation is

$$\frac{d\varepsilon_\theta}{ds} = \frac{e^{\varepsilon_\phi - \varepsilon_\theta}\cos\phi - 1}{s} \quad (3)$$

Since the stress in the thickness direction is negligible compared to the stresses in the plane of the sheet the flow theory can be written as follows :

$$\frac{\dot{\varepsilon}_\theta}{(1+R)\sigma_\theta - R\sigma_\phi} = \frac{\dot{\varepsilon}_\phi}{(1+R)\sigma_\phi - R\sigma_\theta} \quad (4)$$

and

$$\frac{\dot{\varepsilon}_\theta}{(1+R)\sigma_\theta - R\sigma_\phi} = \frac{\dot{\varepsilon}_t}{-(\sigma_\theta + \sigma_\phi)} \quad (5a)$$

Instead of equation (5a) the following equation can be used which includes ε_t and is given from the incompressibility condition.

$$t = t_0\, e^{-(\varepsilon_\theta + \varepsilon_\phi)} \quad (5)$$

Hill's [4] anisotropic yield criterion and the associated flow rule can be simplified to the following equations.

$$\bar{\sigma} = \left(\sigma_\phi^2 - \frac{2R}{1+R}\sigma_\theta\sigma_\phi + \sigma_\phi^2\right)^{1/2} \quad (6)$$

and

$$\dot{\bar{\varepsilon}} = \frac{1+R}{\sqrt{1+2R}}\left(\dot{\varepsilon}_\phi^2 + \frac{2R}{1+R}\dot{\varepsilon}_\phi\dot{\varepsilon}_\theta + \dot{\varepsilon}_\theta^2\right)^{1/2} \quad (7)$$

The strain hardening characteristic used is of the form

$$\sigma/K = \varepsilon^n \quad (8a)$$

because this type of equation will enable the effect of R to be visualized closely for a given value of n and K. But due to the difficulty in starting the numerical solution the following modified form is used for the numerical solution.

$$\sigma/K = (0.0015 + \varepsilon)^n \quad (8)$$

This type of equation will not affect the result specially when the strain is not very low.

The initial conditions (at the beginning of deformation) are

$$\varepsilon_\phi = \varepsilon_\theta = \bar{\varepsilon} = \phi = 0 \;, \quad t = t_0 \;(\text{i.e. } \varepsilon_t = 0)$$

$$\bar{\sigma}/K = (0.0015)^n \;; \quad \sigma_\theta/K = \sigma_\phi/K = \frac{\sqrt{1+R}}{2}(0.0015)^n$$

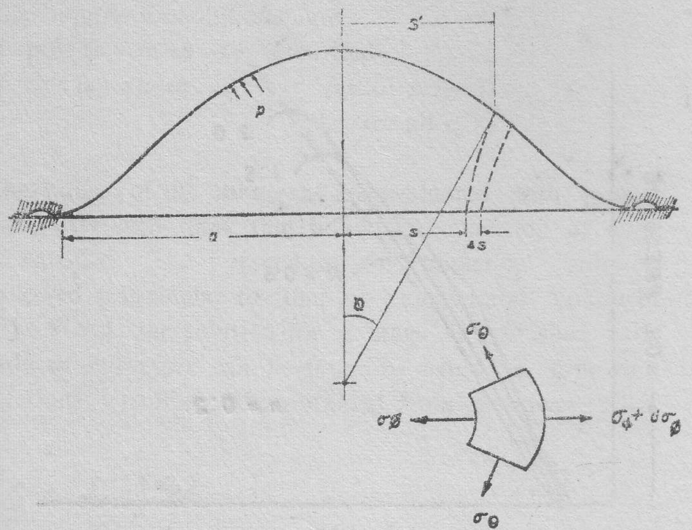

FIGURE 1 : HYDROSTATIC BULGING.

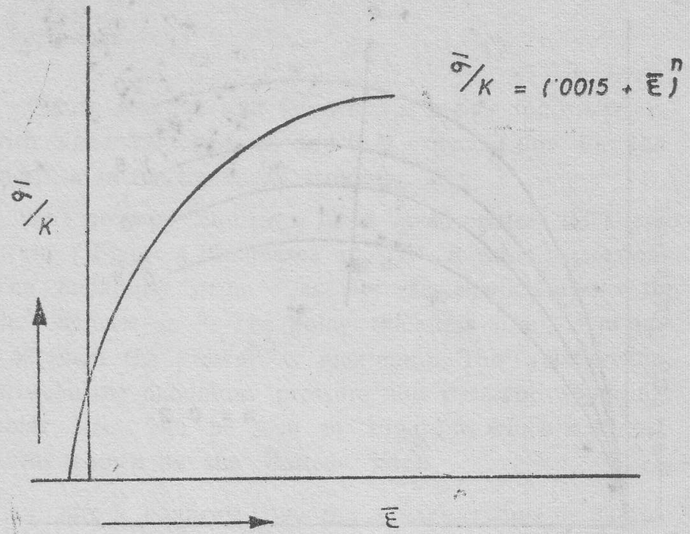

FIGURE 2 : STRESS-STRAIN RELATIONSHIP USED FOR THE NUMERICAL SOLUTION

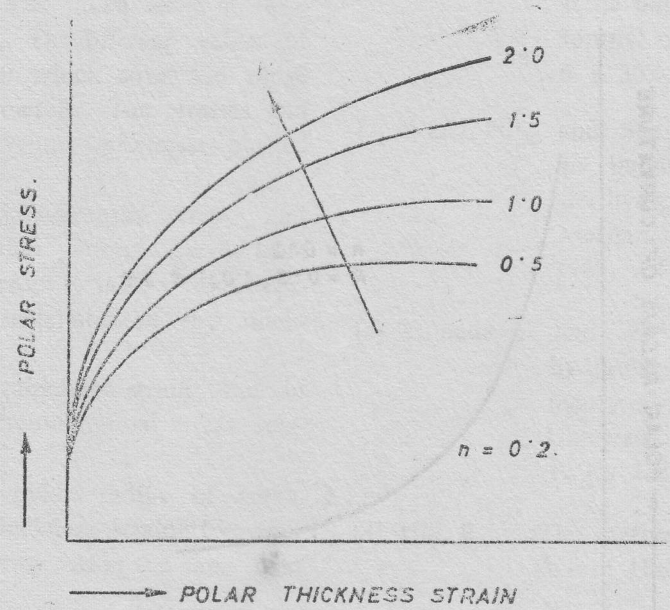

FIGURE : 3 : EFFECT OF R ON THE STRESS - STRAIN CHARACTERISTIC

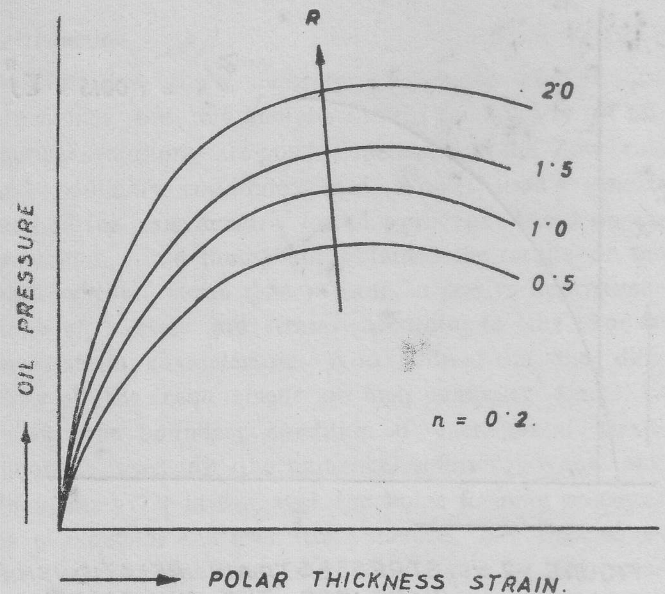

FIGURE 4. EFFECT OF R ON PRESSURE - THICKNESS STRAIN RELATIONSHIP AT POLE.

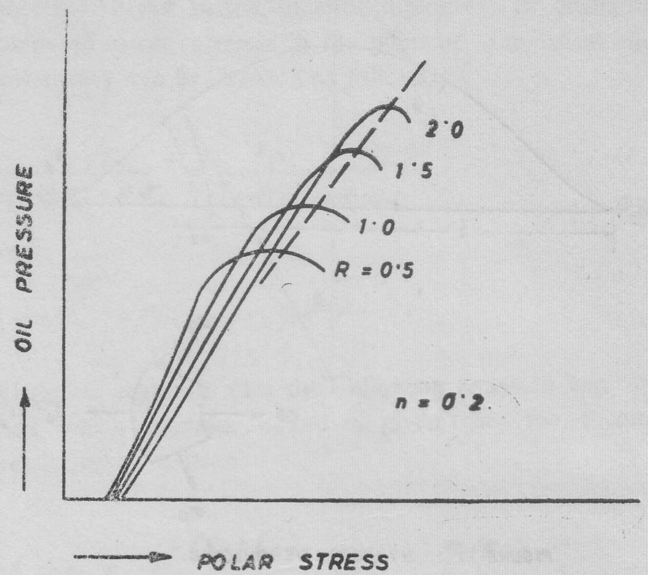

FIGURE 5 : EFFECT OF R ON PRESSURE-POLAR STRESS RELATIONSHIP.

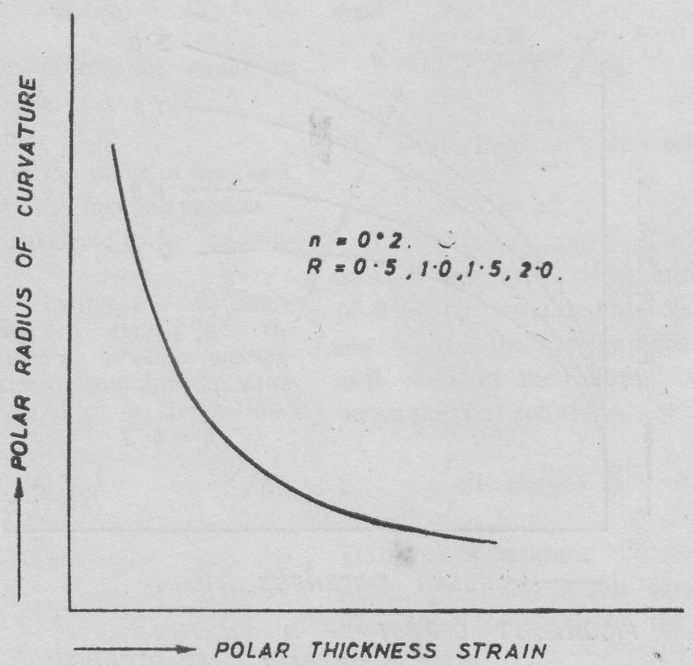

FIGURE 6 : RELATION BETWEEN POLAR RADIUS OF CURVATURE AND POLAR THICKNESS STRAIN FOR DIFFERENT VALUES OF R

The boundary conditions are
at pole (s=0), $\varepsilon\theta=\varepsilon\phi$, $\phi=0$ and
at edge (s=a). $\varepsilon\theta=0$ (at all stages).
i.e. for all ε_0

The values of the unknowns are evaluated with p as a parameter such that the boundary condition at edge is satisfied. The procedure for numerical solution followed was similar to that of Yamada and Yokouchi [3]. When the solution for a stage is finished the radii of curvature can be found by using the following relations which can be obtained from geometry.

$$\rho_2 = \frac{2t(K\sigma_\phi)}{p} \qquad (9)$$

$$\rho_1 = \frac{p\,\rho_2^2}{2p\rho_2 - t(\sigma_\theta K)} \qquad (10)$$

Results

The numerical solutions are shown in Figures 3 to 6. The following values (typical of Aluminium killed steel which was tested by tension) were used : K=71,900; t_0 =0.036 in. The radius 'a' was taken as 5 in. The value of n was taken as 0.2. The different values of R were 0.5, 1.0 , 1.5 and 2.0 which cover the range of ordinary industrial sheet metals. The stresses and radii were dimensionless. The values of stresses plotted are of the order of σ/K.

In Figure 3, the polar circumferential stress and polar thickness strain relationship patterns have been shown for materials with different R values. These curves give the stress strain characteristics in the thickness direction.

In Figure 4, pressure-polar thickness strain and in Figure 5, the pressure-polar circumferential stress relationships have been plotted.

In Figure 6, dimensionless polar radius of curvature (ρ/a) against the polar thickness strain for four different materials have been drawn within the same axes.

Conclusions

Figure 3 shows that the stress is higher for materials with higher values of R and it is expected due to the increase in the thickness strength.

The pressure required for a given polar thickness strain (Figure 4) increases if the R-value increases. The instability strain does not vary significantly with the increase in R. The polar thickness strain is about 0.68 when the pressure is maximum. The relationship between the maximum pressure and the corresponding polar stress can be seen in Figure 5 which is almost linear (shown by the dotted line).

Figure 6 confirms that the polar radius of curvature does not change due to change in R values which implies that the geometry of the bulge is less sensitive to the value of R of the metal.

References

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